HP-35s Calculator Program TM 2

Convert Transverse Mercator Co-ordinates (UTM, SPCS, etc.) to Latitude and Longitude

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Date: April, 2008. **Version**: 1.0 **Mnemonic**: **Y** for TM X & **Y** to Lat/Long

Line	Instruction	Display	User Instructions
Y001	LBL Y		↱ LBL Y
Y002	CLSTK		↱ CLEAR 5
Y003	FS? 10		↰ FLAGS 3 .0
Y004	GTO Y008		
Y005	SF 1		↰ FLAGS 1 1
Y006	SF 10		↰ FLAGS 1 .0
Y007	GTO Y009		
Y008	CF 1		↰ FLAGS 2 1
Y009	TM 2 LAT–LONG		(Key in using EQN RCL T, RCL M, etc.)
Y010	PSE		↱ PSE
Y011	CL x		↱ CLEAR 1
Y012	STO X		↱ STO X
Y013	STO Y		↱ STO Y
Y014	6378137		a value for ellipsoid (WGS84)
Y015	STO A		↱ STO A
Y016	0.00669438		e^2 value for ellipsoid (WGS84)
Y017	STO E		↱ STO E
Y018	0.9996		k_0 value for zone (UTM 17)
Y019	STO K		↱ STO K
Y020	–81		λ_0 for zone (UTM 17)
Y021	STO D		↱ STO D
Y022	0		ϕ_0 for zone (UTM 17)
Y023	STO C		↱ STO C
Y024	500000		E_0 for zone (UTM 17)
Y025	STO I		↱ STO I
Y026	0		N_0 for zone (UTM 17)
Y027	STO J		↱ STO J
Y028	CHECK–ENTER A		(Key in using EQN RCL C, RCL H, etc.)
Y029	PSE		↱ PSE
Y030	INPUT A		↰ INPUT A
Y031	CHECK–ENTER E		(Key in using EQN RCL C, RCL H, etc.)
Y032	PSE		↱ PSE
Y033	INPUT E		↰ INPUT E
Y034	CHECK–ENTER K		(Key in using EQN RCL C, RCL H, etc.)
Y035	PSE		↱ PSE
Y036	INPUT K		↰ INPUT K
Y037	CHK–NTR LONG 0		(Key in using EQN RCL C, RCL H, etc.)
Y038	PSE		↱ PSE

Transverse Mercator Co-ordinates to Latitude/Longitude

Line	Instruction
Y039	INPUT D
Y040	CHK–NTR LAT 0
Y041	PSE
Y042	INPUT C
Y043	CHK–NTR E 0
Y044	PSE
Y045	INPUT I
Y046	CHK–NTR N 0
Y047	PSE
Y048	INPUT J
Y049	RCL D
Y050	HMS→
Y051	STO D
Y052	RCL C
Y053	HMS→
Y054	STO C
Y055	ENTER EASTING
Y056	PSE
Y057	INPUT X
Y058	ENTER NORTHING
Y059	PSE
Y060	INPUT Y
Y061	RCL I
Y062	STO— X
Y063	RCL J
Y064	STO— Y
****	Compute b
Y065	1
Y066	RCL— E
Y067	√x
Y068	RCL× A
Y069	STO B
****	Compute n
Y070	RCL A
Y071	RCL— B
Y072	RCL A
Y073	RCL+ B
Y074	÷
Y075	STO O
****	Compute ΔN
Y076	RCL Y
Y077	RCL÷ K
Y078	STO M
****	Compute m_0 and m
Y079	RCL C

Line	Instruction
Y080	→RAD
Y081	CF 10
Y082	(1 – E ÷ 4 – 3 × E^2 ÷ 64 – 5 × E^3 ÷ 256)
Y083	×
Y084	STO V
Y085	0.375 × (E + E^2 ÷ 4 + 15 × E^3 ÷ 128) × SIN(2 × C)
Y086	STO— V
Y087	15 ÷ 256 × (E^2 + 0.75 × E^3) × SIN(4 × C)
Y088	STO+ V
Y089	35 × E^3 ÷ 3072 × SIN(6 × C)
Y090	STO— V
Y091	RCL A
Y092	RCL× V
Y093	STO+ M
****	Compute G and σ
Y094	RAD [Key in as MODE 2]
Y095	(1 + 2.25 × O^2 + 225× O^4 ÷ 64) × (1 – O) × (1 – O^2)
Y096	RCL× A
Y097	RCL M
Y098	x < > y
Y099	÷
Y100	STO S
****	Compute ϕ' (the foot-point latitude)
Y101	RCL S
Y102	STO F
Y103	(1.5 × O – 25 × O^3 ÷ 32) × SIN(2 × S)
Y104	STO+ F
Y105	(21 × O^2 ÷ 16 – 55 × O^4 ÷ 32) × SIN(4 × S)
Y106	STO+ F
Y107	151 × O^3 ÷ 96 × SIN(6 × S)
Y108	STO+ F
Y109	1097 × O^4 ÷ 512 × SIN(8 × S)
Y110	STO+ F
****	Compute ν', t', ρ', ψ' and x
Y111	RCL F
Y112	TAN
Y113	STO T
Y114	A ÷ SQRT(1 — E × SIN(F)^2)
Y115	STO N
Y116	A × (1 — E) ÷ ((1 — E × SIN(F)^2)^1.5)
Y117	STO R
Y118	N ÷ R
Y119	STO P
Y120	X ÷ N ÷ K
Y121	STO U

Transverse Mercator Co-ordinates to Latitude/Longitude

Line	Instruction
****	Compute ϕ (latitude)
Y122	U × T × X ÷ K ÷ R ÷ 2
Y123	STO— F
Y124	(12 × T^2 + 9 × P × (1 — T^2) — 4 × P^2) × U^3 × X × T ÷ K ÷ R ÷ 24
Y125	STO+ F
Y126	(8 × P^4 × (11 – 24 × T^2) – 12 × P^3 × (21 – 71 × T^2) + 15 × P^2 × (15 – 98 × T^2 +
†	… 15 × T^4) + 180 × P × (5 × T^2 – 3 × T^4) + 360 × T^4) × X × T ÷ K ÷ R ÷ 720
Y127	RCL U
Y128	5
Y129	y^x
Y130	×
Y131	STO— F
Y132	(1385 + 3633 × T^2 + 4095 × T^4 + 1575 × T^6) × X × T ÷ K ÷ R ÷ 40320 × U^7
Y133	STO+ F
****	Calculate λ (longitude)
Y134	1
Y135	STO W
Y136	(P + 2 × T^2) × U^2 ÷ 6
Y137	STO— W
Y138	(24 × T^4 + 72 × P × T^2 + P^2 × (9 – 68 × T^2) – 4 × P^3 × (1 – 6 × T^2)) × U^4 ÷ 120
Y139	STO+ W
Y140	(61 + 662 × T^2 + 1320 × T^4 + 720 × T^6) × U^6 ÷ 5040
Y141	STO— W
Y142	RCL U
Y143	STO× W
Y144	RCL T
Y145	ATAN
Y146	COS
Y147	STO÷ W
****	Calculate γ (grid convergence)
Y148	—1
Y149	STO G
Y150	(T^2 + 3 × P – 2 × P^2) × U^2 ÷ 3
Y151	STO+ G
Y152	(P^4 × (11 – 24 × T^2) – 3 × P^3 × (8 – 23 × T^2) + 5 × P^2 × (3 – 14 × T^2) + 30 × P
†	… × T^2 + 3 × T^4) × U^4 ÷ 15
Y153	STO— G
Y154	(17 + 77 × T^2 + 105 × T^4 + 45 × T^6) × U^8 ÷ 315
Y155	STO+ G
Y156	RCL T
Y157	RCL× U
Y158	STO× G
****	Calculate k (point scale factor)
Y159	RCL X
Y160	STO× U

Transverse Mercator Co-ordinates to Latitude/Longitude

Line	Instruction
Y161	RCL K
Y162	STO÷ U
Y163	RCL R
Y164	STO÷ U
Y165	((4 × P × (1 – 6 × T^2) – 3 × (1 – 16 × T^2) – 24 × T^2 ÷ P) × U^2 ÷ 24 + U ÷ 2 + 1 +
†	… U^3 ÷ 720) × K
Y166	STO S

Line	Instruction
****	Prepare results
Y167	DEG [Key in as MODE 1]
Y168	SF 10
Y169	RCL F
Y170	→DEG
Y171	→HMS
Y172	STO F
Y173	RCL W
Y174	→DEG
Y175	RCL+ D
Y176	→HMS
Y177	STO L
Y178	RCL G
Y179	→DEG
Y180	→HMS
Y181	STO G
****	Show results
Y182	RESULTS
Y183	PSE
Y184	LATITUDE
Y185	PSE
Y186	VIEW F
Y187	LONGITUDE
Y188	PSE
Y189	VIEW L
Y190	GRID CONV
Y191	PSE

Line	Instruction
Y192	VIEW G
Y193	PT SCALE FACT
Y194	PSE
Y195	VIEW S
Y196	0
Y197	STO Q
Y198	NEXT PT [0—1]
Y199	PSE
Y200	INPUT Q
Y201	RCL Q
Y202	x = 0?
Y203	GTO Y213
Y204	NEW ZONE [0—1]
Y205	PSE
Y206	0
Y207	STO Q
Y208	INPUT Q
Y209	RCL Q
Y210	x = 0?
Y211	GTO Y055
Y212	GTO Y028
Y213	PROGRAM END
Y214	PSE
Y215	FS? 1
Y216	CF 10
Y217	RTN

Notes

(1) The program should be run in RPN mode, as results in ALG mode are unknown.

(2) Latitudes and longitudes should be entered in HP notation, i.e., DDD.MMSS. The grid convergence is displayed in HP notation.

(3) The program may be used for any Transverse Mercator projection, if the appropriate parameters are known. Similarly, any ellipsoid may be used, if its a and e^2 parameters are known. Parameters for a wide range of ellipsoids, all UTM zones and all SPCS TM zones are included at the end of this document.

(4) Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in North America. It is critical to enter the correct sign in the calculator when entering values.

(5) Lines with **** are comments only, and should not be entered into the calculator. They are there to make program entry a little easier.

(6) This program is long and often appears to be a stream of meaningless commands. This means that it may be more prone to errors when being entered. It is suggested that the program be entered using the given constants, tested (and the checksum checked), and when it is satisfactory, the constants at the start of the program can be changed to those most suitable for the bulk of the expected work.

(7) In order to reduce the apparent length of the program (which otherwise would have been well over 600 lines), equations were used for the bulk of the computations. Some equations are too long to fit on a single line in the above listing, so are continued to the line below (Y126, Y152, Y165. In this case, the line number is replaced by a † and a … appears at the start of the continuing line. Neither the † nor the … should be entered into the calculator.

The use of equations, rather than direct instruction code, does slow the computation process a little, but makes the program a bit shorter and so possibly easier to enter.

(8) The program allows the user to run additional points after each is completed, by prompting. If another point is to be processed, the user also has the option to move to a new zone and ellipsoid, otherwise the previous ellipsoid and projection parameters are used. Respond 0 for 'NO' and 1 for 'YES' at the Q? prompt. If the user chooses to enter another point, the previous data entered is displayed at the prompts, but angular data are stored in decimal degrees. This should be re-entered in HP notation (or quickly converted with the ↱ →HMS key sequence), even if the same data is being used, because the program will convert the values to decimal degrees again, and so produce erroneous results.

(9) There are two lines where the calculator's mode is changed from DEGREES to RADIANS, and *vice versa*. The instructions for these lines (Y094 and Y167) are keyed in using the MODE button. A note about this is included on the relevant line of the code, in red, to avoid confusion with the →RAD and →DEG function, which are used elsewhere in the program.

(10) The program appears to work correctly, as tested. However, the grid convergence result has the opposite sign to that produced by the NGS on-line Lat/Long to SPCS conversion package at: http://www.ngs.noaa.gov/cgi-bin/spc_getpc.prl The formulae are correct in this program, and the results agree with the sign convention of Redfearn's formulae, as well as the normal usage of the grid convergence (converting between grid and true azimuths). I am not sure why the NGS program has the opposite sign, but I have asked NGS about it. Until this difference is resolved, be aware that the sign could be the opposite, and work out the correct sign from first principles.

Transverse Mercator Co-ordinates to Latitude/Longitude

Theory

Converting from cartesian co-ordinates (E, N or X, Y) to geographical co-ordinates (latitude and longitude) on a Transverse Mercator projection is a straightforward transformation, if somewhat long-winded. This program uses equations to help reduce the bulk of the program a little.

Given that we have a, e^2, ϕ, k_0, λ and λ_0, we can use the following expressions for the conversion. These are Redfearn's Formulae. Note that these use an extra term in the computations, compared to Snyder's book (1987), but this will make a negligible difference in the overall values. The results will be a little different to the tabulated values for SPCS, too, owing to the limitations on the SPCS 27 computations. Remembering that the allowable distortion in the SPCS was to be no more than 1 in 10,000, it is acceptable to drop the final term in the formulae, as this doesn't degrade the formulae by anywhere near 1 in 10,000. Such modified formulae will then agree with Snyder's formulae, remembering that Snyder was setting up the formulae for mapping, rather than geodetic use.

For UTM computations, you should use the full number of terms. This is because there is no 'legal' tolerance of distortion in the conversion process. UTM co-ordinates are now printed on 1:24,000 quadrangle maps, with either a grid/graticule or marginal ticks. These UTM co-ordinates are often on the NAD27 datum and need to be converted to NAD83 before they can be used. While there is a marginal note concerning the conversion of latitude and longitude from NAD27 to NAD83 on many of the more recent mapsheets, this value **does not** apply to the UTM co-ordinates (or the SPCS co-ordinates). This is because the latitude and longitude values are, in effect, figured from the origin in Kansas, while the UTM Northing co-ordinates are figured from the Equator. SPCS northings are figured from the zone origin, so will have a different shift for each zone. You should convert the co-ordinates to latitude and longitude for the appropriate system, convert these to NAD83, then convert to UTM or SPCS TM co-ordinates. An approximate set of shifts for UTM can be found in a paper by Welch, R., and Homsey, A., "Datum Shifts for UTM Co-ordinates," in the *Photogrammetric Engineering and Remote Sensing* journal, Volume 63, No. 4, pp. 371–375, published in 1997.

Conversion Formulae

Starting with the following general formulae, these can be applied in the following conversion formulae.

$E' = E - E_0$ (E_0 is the offset of the central meridian; check the value for each zone. For UTM, E_0 = 500 000·000 meters.)

$N' = N - N_0$ (N_0 is the offset of the origin latitude; check the value for each zone. For UTM in the northern hemisphere, $N_0 = 0$; for UTM in the southern hemisphere, N_0 = 10 000 000·000 meters.)

ν = radius of curvature in the prime vertical at ϕ; i.e. $\nu = \dfrac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$

$$\rho = \frac{a\,(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}}$$ = radius of curvature in the meridian at ϕ

$$\omega = \lambda - \lambda_0$$

$$\psi = \frac{\nu}{\rho}$$ i.e. ratio of the radii of curvature at ϕ

$t \;=\; \tan\phi$

m = meridian distance from equator, computed using the following expression

$$m \;=\; a\,(A_0\,\phi \;-\; A_2 \sin 2\phi \;+\; A_4 \sin 4\phi \;-\; A_6 \sin 6\phi)$$

where ϕ is in radians and

$$A_0 \;=\; 1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256}$$

$$A_2 \;=\; \frac{3}{8}\left(e^2 + \frac{e^4}{4} + \frac{15e^6}{128}\right)$$

$$A_4 \;=\; \frac{15}{256}\left(e^4 + \frac{3e^6}{4}\right)$$

$$A_6 \;=\; \frac{35e^6}{3072}$$

With the appropriate values for ellipsoids and scale factors, these formulae will work for any Transverse Mercator projection: UTM, SPCS, AMG, MGA or whatever.

In order to convert the given northing to latitude, we first need to calculate what is known as the foot-point latitude, ϕ', which is the latitude for which the meridian distance is equal to $\frac{N'}{k_0}$. This value can be calculated directly provided three other values, namely n, G and σ are calculated first. The choice of variable names for these three values is historical and isn’t related to any other use of them.

$$n \;=\; \frac{a - b}{a + b}$$ where a and b are the semi-major and semi-minor axes

$$b = a\,\sqrt{1 - e^2}$$

$$G \;=\; a\,(1 - n)\,(1 - n^2)\,\left(1 + \frac{9}{4}\,n^2 + \frac{225}{64}\,n^4\right)\frac{\pi}{180}$$

= mean length of an arc of one degree of the meridian

$$\sigma \;=\; \frac{m\,\pi}{180\,G} \qquad \text{use } m \;=\; \frac{N'}{k_0}$$

$$\begin{aligned}\phi' \;=\; \sigma \quad &+ \left(\frac{3n}{2} - \frac{27n^3}{32}\right)\sin 2\sigma \\ &+ \left(\frac{21n^2}{16} - \frac{55n^4}{32}\right)\sin 4\sigma \\ &+ \left(\frac{151n^3}{96}\right)\sin 6\sigma \\ &+ \left(\frac{1097n^4}{512}\right)\sin 8\sigma\end{aligned}$$

Transverse Mercator Co-ordinates to Latitude/Longitude

With these values we can calculate the geographical co-ordinates directly. Note that t', ψ', ρ' and ν' are functions of the foot-point latitude and using the same formulae as listed above.

Latitude (in radians)

$$\text{Let } x = \frac{E'}{k_0\, \nu'}$$

$$\begin{aligned}\phi = \phi' & - \frac{t'}{k_0\, \rho'}\ x\ \frac{E'}{2} \\ & + \frac{t'}{k_0\, \rho'}\ \frac{x^3\, E'}{24}\ [-4\, \psi'^2 + 9\, \psi'\, (1 - t'^2) + 12\, t'^2] \\ & - \frac{t'}{k_0\, \rho'}\ \frac{x^5\, E'}{720}\ [8\, \psi'^4\, (11 - 24\, t'^2) - 12\, \psi'^3\, (21 - 71\, t'^2) + 15\, \psi'^2\, (15 - 98\, t'^2 + 15\, t'^4) \\ & \qquad + 180\, \psi'\, (5\, t'^2 - 3\, t'^4) + 360\, t'^4] \\ & + \frac{t'}{k_0\, \rho'}\ \frac{x^7\, E'}{40320}\ (1385 + 3633\, t'^2 + 4095\, t'^4 + 1575\, t'^6)\end{aligned}$$

Longitude (in radians)

$$\text{Let } x = \frac{E'}{k_0\, \nu'}$$

$$\begin{aligned}\omega = & \sec\phi'\ x \\ & - \sec\phi'\ \frac{x^3}{6}\ (\psi' + 2\, t'^2) \\ & + \sec\phi'\ \frac{x^5}{120}\ [-4\, \psi'^3\, (1 - 6\, t'^2) + \psi'^2\, (9 - 68\, t'^2) + 72\, \psi'\, t'^2 + 24\, t'^4] \\ & - \sec\phi'\ \frac{x^7}{5040}\ (61 + 662\, t'^2 + 1320\, t'^4 + 720\, t'^6)\end{aligned}$$

Grid Convergence (in radians)

$$\text{Let } x = \frac{E'}{k_0\, \nu'}$$

$$\begin{aligned}\gamma = & - t'\, x \\ & + t'\ \frac{x^3}{3}\ (-2\, \psi'^2 + 3\, \psi' + t'^2) \\ & - t'\ \frac{x^5}{15}\ [\psi'^4\, (11 - 24\, t'^2) - 3\, \psi'^3\, (8 - 23\, t'^2) + 5\, \psi'^2\, (3 - 14\, t'^2) + 30\, \psi'\, t'^2 + 3\, t'^4] \\ & + t'\ \frac{x^7}{315}\ (17 + 77\, t'^2 + 105\, t'^4 + 45\, t'^6)\end{aligned}$$

Transverse Mercator Co-ordinates to Latitude/Longitude

Point Scale Factor

Let $x = \dfrac{E'^2}{k_0^2\, \nu'\, \rho'}$ (note the different value of x for this formula)

$$k = k_0 \left(1 + \frac{x}{2} + \frac{x^2}{24}\left(4\,\psi'\,(1 - 6\,t'^2) - 3\,(1 - 16\,t'^2) - \frac{24\,t'^2}{\psi'}\right) + \frac{x^3}{720} \right)$$

Given these values, the latitude and grid convergence are converted to degrees, minutes and seconds, while ω is converted to the longitude, λ, using the formula below, and then converted to degrees, minutes and seconds.

$$\lambda = \omega + \lambda_0$$

Sample Computations

Example 1

Using the SPCS 1983 ($a = 6{,}378{,}137$ m, $e^2 = 0.006\,694\,3800$), the following results are obtained.

Nevada East Zone, 2701, $\lambda_0 = -115° \, 35'$, $\phi_0 = 34° \, 45'$, $k_0 = 0.999\,900$, $E_0 = 200{,}000.000$ m, $N_0 = 8{,}000{,}000.000$ m.

Easting (E) = 185,603.123 m Northing (N) = 8,739,929.417 m

Latitude = 41° 25' 00".000 Longitude = –115° 45' 20".000

Grid Convergence (γ) = 0° 06' 50.1" Point Scale Factor (k) = 0.999 902 55

Example 2

Using the SPCS 1927 ($a = 20925832.2$ ft, $e^2 = 0.006\,768\,66$), the following results are obtained.

Nevada East Zone, SPCS 1927, $\lambda_0 = -115° \, 35'$, $\phi_0 = 34° \, 45'$, $k_0 = 0.999\,900$, $E_0 = 500{,}000.000$ ft, $N_0 = 0.000$ ft.

Easting (E) = 452,764.960 ft Northing (N) = 2,427,533.222 ft

Latitude = 41° 25' 00".000 Longitude = –115° 45' 20".000

Grid Convergence (γ) = 0° 06' 50.1" Point Scale Factor (k) = 0.999 902 55

Example 3

Using the ANS ellipsoid ($a = 6{,}378{,}160$ m, $e^2 = 0.006\,694\,541\,855$), the following results are obtained.

AMG Zone 54, $\lambda_0 = +141° \, 00'$, $\phi_0 = 0° \, 00'$, $k_0 = 0.999\,600$,
$E_0 = 500{,}000.000$ m, $N_0 = 10{,}000{,}000.000$ m.

Easting (E) = 758,053.090 m Northing (N) = 5,828,496.973 m

Latitude = –37° 39' 15".557 Longitude = +143° 55' 30".6330

Grid Convergence (γ) = +1° 47' 16.67" Point Scale Factor (k) = 1.000 420 30

Example 4

Using the WGS-72 ellipsoid (a = 6,378,135 m, e^2 = 0.006 694 317 778), the following results are obtained.

UTM Zone 58, λ_0 = +165° 00', ϕ_0 = 0° 00', k_0 = 0.999 600,
E_0 = 500,000.000 m, N_0 = 10,000,000.000 m.

Easting (E) = 758,053.090 m Northing (N) = 5,828,496.973 m

Latitude = –37° 39' 15".557 Longitude = +143° 55' 30".6330

Grid Convergence (γ) = +1° 47' 16.67" Point Scale Factor (k) = 1.000 420 30

These last two sets of results agree with those computed in the AGD Technical Manual, 1986.

Running the Program

Press XEQ Y , then the ENTER key, to start the program. The calculator briefly displays TM 2 LAT—LONG, then briefly shows CHECK—ENTER A. This is "Point A," discussed below. The program then stops and displays the prompt for entering the semi-major axis value, while displaying the current default value:

A?
6,378,137.0000 (This is for GRS80/WGS84/NAD83)

If you are happy with this value for the semi-major axis of the ellipsoid, press R/S to continue. Otherwise. Key in a different value (for a different ellipsoid, e.g., 6378135 for WGS72) and press R/S to continue. (This discussion will use the data from Example 4, in the Sample Computations section, above.)

The calculator briefly displays CHECK—ENTER E. The program then stops and displays the prompt for entering the eccentricity of the ellipsoid, e:

E?
0.00669438 (This is for GRS80/WGS84/NAD83)

If this value for the eccentricity is correct, press R/S to continue. Otherwise, key in a different value (for a different ellipsoid, e.g., 0.006 694 317 778 for WGS-72) and press R/S to continue.

The calculator briefly displays CHECK—ENTER K. The program then stops and displays the prompt for entering the scale factor at the central meridian (λ_0), which is k_0:

K?
0.9996000 (This is for UTM)

If this value for the scale factor is satisfactory, press R/S to continue. If you want to change it, such as for an SPCS zone, key in the correct value and press R/S.

The calculator briefly displays CHK—NTR LONG 0. The program then stops and displays the prompt for entering the longitude of the central meridian of the projection, λ_0. Note that in the western hemisphere, this will be a negative value, and should be in HP notation (DDD.MMSS).

D?
—81.000000 (This is for UTM Zone 17)

If this is the correct central meridian, press R/S to continue, if this is not correct, key in the correct value, in HP notation, then press R/S to continue. In this case, key in 165 for UTM Zone 58, then press R/S.

The calculator briefly displays CHK—NTR LAT 0. The program then stops and displays the prompt for entering the latitude of the Northing co-ordinate origin, ϕ_0. For UTM, this is 0.000 (the equator), while for SPCS Zones, it is usually a latitude well south of the zone. The value should be entered in HP notation.

C?
0.000000 (This is for UTM)

If this is the correct latitude base, press R/S to continue. If you want a different value, key in that value and press R/S to continue. In this case, press R/S to continue.

The calculator briefly displays CHK—NTR E 0. The program then stops and prompts for entry of the false easting value, or the easting offset, denoted E_0. This is the value of the easting co-ordinate at the central meridian, λ_0. For UTM, this is 500,000.0000, while its value varies for different SPCS zones.

I?
500,000.0000 (This is for UTM)

If this is the correct value, press R/S to continue. If a different value is desired, key in the value and press R/S. In this case, press R/S to continue.

The calculator briefly displays CHK—NTR N 0. The program then stops and prompts for the false northing value, or the northing offset. This is the value of the northing at the point ϕ_0, λ_0, denoted N_0.

J?
0.0000 (This is for UTM in the northern hemisphere)

If this is the correct value, press R/S to continue. If a different value is required, key in the value and press R/S. In this case, key in 10,000,000.000 and press R/S. This is the N_0 value for UTM in the southern hemisphere.

This is "Point B," discussed below. The calculator briefly displays ENTER EASTING. The program stops and displays the prompt for entering the easting of the point to be converted.

X?
0.0000

Key in the easting of the point and press R/S to continue. In this case, key in 787420.487 and press R/S.

The calculator briefly displays ENTER NORTHING. The program then stops and displays the prompt for entering the northing of the point to be converted.

Y?
0.0000

Key in the northing of the point and press R/S to continue. In this case, key in 6782165.201 and press R/S.

Transverse Mercator Co-ordinates to Latitude/Longitude

The program displays RUNNING for some little time, then displays RESULTS briefly, followed by LATITUDE briefly. The program then stops and displays the latitude value of the point, in HP notation. In this case, the calculator displays:

F=
–29.03231530

This is the latitude of the point, in this case being 29° 03' 23".1530 S in more conventional notation. Press R/S to continue. The calculator briefly displays LONGITUDE, then stops and displays the longitude value of the point, in HP notation. In this case, the calculator displays:

L=
167.57066320

This is the longitude of the point, in this case being 167° 57' 06".6320 E in more conventional notation. Press R/S to continue. The calculator briefly displays GRID CONV, then stops and displays the grid convergence value in HP notation. In this case, the calculator displays:

G=
1.26045907

This is the grid convergence in HP notation, and is 1° 26' 04".59 in more conventional notation. Press R/S to continue. The calculator briefly displays PT SCALE FACT, then stops and displays the point scale factor of the point on the Transverse Mercator projection. In this case, the calculator displays:

S=
1.00061955

This is the point scale factor. Press R/S to continue.

You now have the choice of running one or more additional points. The calculator briefly displays NEXT PT [0—1], then stops and displays the prompt for answering questions:

Q?
0.0000

If you want to quit the program, just press R/S. If you want to enter more points, key in 1 and press R/S. In this case, the calculator then prompts to see if you want to use the same parameters. The calculator briefly displays NEW ZONE [0—1], then stops at the question prompt:

Q?
0.0000

If you want to go to a new zone, key in 1 and press R/S, and the calculator will take you to the point where you can change any of the values (Point A above), starting with the ellipsoid parameters. If you want to work in the same zone already entered, just press R/S, and the program will take you to "Point B" and prompt for the latitude of the point to be converted, and continue from there. You can go around the program as many times as necessary.

When you choose to end the program, the calculator briefly displays PROGRAM END and then comes to an end, returning to the point from which it was called, or to normal operations, and resetting Flag 10.

Transverse Mercator Co-ordinates to Latitude/Longitude

Storage Registers Used

A	Semi-major axis of the ellipsoid being used, a
B	Semi-minor axis of the ellipsoid being used, b
C	ϕ_0, the origin latitude for the co-ordinates
D	λ_0, the central meridian of the projection
E	Eccentricity of the ellipsoid, e^2
F	ϕ', foot-point latitude, then ϕ, latitude of the point that has been converted
G	γ, the grid convergence of the point being converted
H	meridian distance of the origin latitude, ϕ_0
I	E_0, the offset for the eastings (the easting at λ_0)
J	N_0, the offset for the northings (the northing at ϕ_0, λ_0)
K	k_0, the scale factor along the central meridian, λ_0
L	λ, longitude of the point that has been converted
M	m, preliminary meridian distance of the point to be converted
N	ν'
O	n, a constant for the ellipsoid
P	$\psi' = \dfrac{\nu'}{\rho'}$
Q	used for getting responses to questions about running more points
R	ρ
S	σ, an intermediate value; then k, point scale factor at the point being converted
T	$\tan \phi'$
U	$x = E' \div k_0\ \nu'$
V	m_o, the meridian distance to ϕ_0
W	$\omega = \lambda - \lambda_0$
X	Easting co-ordinate of point to be converted
Y	Northing co-ordinate of point to be converted

Statistical Registers: not used

Labels Used

Label **Y** Length = 1707 Checksum = 72F1

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computations to check proper operation after entry.

Flags Used

Flags 1 and 10 are used by this program. Flag 10 is set for this program, so that equations can be shown as prompts. Flag 1 is used to record the setting of Flag 10 before the program begins. At the end of the program, Flag 10 is reset to its original value, based on the value in Flag 1.

Parameters for the Computations

Universal Transverse Mercator (UTM)

For UTM, the ϕ_0 value is 0° (the equator) for both northern and southern hemispheres. The λ_0 values are given for each zone in the table below.

Zone	Central Meridian, λ_0	Zone	Central Meridian, λ_0
1	177° W	31	3°E
2	171° W	32	9°E
3	165° W	33	15°E
4	159° W	34	21°E
5	153° W	35	27°E
6	147° W	36	33°E
7	141° W	37	39°E
8	135° W	38	45°E
9	129° W	39	51°E
10	123° W	40	57°E
11	117° W	41	63°E
12	111° W	42	69°E
13	105° W	43	75°E
14	99° W	44	81°E
15	93° W	45	87°E
16	87° W	46	93°E
17	81° W	47	99°E
18	75° W	48	105°E
19	69° W	49	111°E
20	63° W	50	117°E
21	57° W	51	123°E
22	51° W	52	129°E
23	45° W	53	135°E
24	39° W	54	141°E
25	33° W	55	147°E
26	27° W	56	153°E
27	21° W	57	159°E
28	15° W	58	165°E
29	9° W	59	171°E
30	3° W	60	177°E

The E_0 value for all zones is 500,000.000 meters. The N_0 value for the northern hemisphere is 0.000 meters. The N_0 value for the southern hemisphere is 10,000,000.000 meters.

State Plane Co-ordinate System (SPCS) 1983

Several US states use the Transverse Mercator projection for SPCS 1983. The various parameters for each zone in the 1983 system are given in the table below. Use these parameters with the program, together with the GRS80/WGS84/NAD83 ellipsoid parameters, in meters.

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
Alabama					
East	85° 50'	30° 30'	0.9999600	200000.00	0.00
West	87° 30'	30° 00'	0.9999333	600000.00	0.00
Alaska					
2	142° 00'	54° 00'	0.9999000	500000.00	0.00
3	146° 00'	54° 00'	0.9999000	500000.00	0.00
4	150° 00'	54° 00'	0.9999000	500000.00	0.00
5	154° 00'	54° 00'	0.9999000	500000.00	0.00
6	185° 00'	54° 00'	0.9999000	500000.00	0.00
7	162° 00'	54° 00'	0.9999000	500000.00	0.00
8	166° 00'	54° 00'	0.9999000	500000.00	0.00
9	170° 00'	54° 00'	0.9999000	500000.00	0.00
Arizona					
East	110° 10'	31° 00'	0.9999000	213360.00	0.00
Central	111° 55'	31° 00'	0.9999000	213360.00	0.00
West	113° 45'	31° 00'	0.9999333	213360.00	0.00
Delaware	72° 25'	38° 00'	0.9999950	200000.00	0.00
Florida					
East	81° 00'	24° 20'	0.9999412	200000.00	0.00
West	82° 00'	24° 20'	0.9999412	200000.00	0.00
Georgia					
East	82° 10'	30° 00'	0.9999000	200000.00	0.00
West	84° 10'	30° 00'	0.9999000	700000.00	0.00
Hawaii					
1	155° 30'	18° 50'	0.9999667	500000.00	0.00
2	156° 40'	20° 20'	0.9999667	500000.00	0.00
3	158° 00'	21° 10'	0.9999900	500000.00	0.00
4	159° 30'	21° 50'	0.9999900	500000.00	0.00
5	160° 10'	21° 40'	1.0000000	500000.00	0.00

Transverse Mercator Co-ordinates to Latitude/Longitude

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
Idaho					
East	112° 10'	41° 40'	0.9999474	200000.00	0.00
Central	114° 00'	41° 40'	0.9999474	500000.00	0.00
Illinois					
East	88° 20'	36° 40'	0.9999750	300000.00	0.00
West	90° 10'	36° 40'	0.9999412	700000.00	0.00
Indiana					
East	85° 40'	37° 30'	0.9999667	100000.00	250000.00
West	87° 05'	37° 30'	0.9999667	900000.00	250000.00
Maine					
East	68° 30'	43° 40'	0.9999000	300000.00	0.00
West	70° 10'	42° 50'	0.9999667	900000.00	0.00
Mississippi					
East	88° 50'	29° 30'	0.9999500	300000.00	0.00
West	90° 20'	29° 30'	0.9999500	700000.00	0.00
Missouri					
East	90° 30'	35° 50'	0.9999333	250000.00	0.00
Central	92° 30'	35° 50'	0.9999333	500000.00	0.00
West	94° 30'	36° 10'	0.9999412	850000.00	0.00
Nevada					
East	115° 35'	34° 45'	0.9999000	200000.00	8000000.00
Central	116° 40'	34° 45'	0.9999000	500000.00	6000000.00
West	118° 35'	34° 45'	0.9999000	800000.00	4000000.00
New Hampshire	71° 40'	42° 30'	0.9999667	300000.00	0.00
New Jersey	74° 30'	38° 50'	0.9999000	150000.00	0.00
New Mexico					
East	104° 20'	31° 00'	0.9999091	165000.00	0.00
Central	106° 15'	31° 00'	0.9999000	500000.00	0.00
West	107° 50'	31° 00'	0.9999167	830000.00	0.00

Transverse Mercator Co-ordinates to Latitude/Longitude

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
New York					
East	74° 30'	40° 00'	0.9999000	150000.00	0.00
Central	76° 35'	40° 00'	0.9999375	250000.00	0.00
West	78° 35'	40° 00'	0.9999375	350000.00	0.00
Rhode Island	71° 30'	41° 05'	0.9999938	100000.00	0.00
Vermont	72° 30'	42° 30'	0.9999643	500000.00	0.00
Wyoming					
East	105° 10'	40° 30'	0.9999375	200000.00	0.00
East Central	107° 20'	40° 30'	0.9999375	400000.00	100000.00
West Central	108° 45'	40° 30'	0.9999375	600000.00	0.00
West	110° 05'	40° 30'	0.9999375	800000.00	100000.00

State Plane Co-ordinate System (SPCS) 1927

Several US states used the Transverse Mercator projection for SPCS 1927. The various parameters for each zone in the 1927 system are given in the table below. Use these parameters with the program, together with the Clarke 1866 ellipsoid in feet.

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Alabama					
East	85° 50'	30° 30'	0.9999600	500000.00	0.00
West	87° 30'	30° 00'	0.9999333	500000.00	0.00
Alaska					
2	142° 00'	54° 00'	0.9999000	500000.00	0.00
3	146° 00'	54° 00'	0.9999000	500000.00	0.00
4	150° 00'	54° 00'	0.9999000	500000.00	0.00
5	154° 00'	54° 00'	0.9999000	500000.00	0.00
6	185° 00'	54° 00'	0.9999000	500000.00	0.00
7	162° 00'	54° 00'	0.9999000	700000.00	0.00
8	166° 00'	54° 00'	0.9999000	500000.00	0.00
9	170° 00'	54° 00'	0.9999000	600000.00	0.00

Transverse Mercator Co-ordinates to Latitude/Longitude

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Arizona					
East	110° 10'	31° 00'	0.9999000	500000.00	0.00
Central	111° 55'	31° 00'	0.9999000	500000.00	0.00
West	113° 45'	31° 00'	0.9999333	500000.00	0.00
Delaware	72° 25'	38° 00'	0.9999950	500000.00	0.00
Florida					
East	81° 00'	24° 20'	0.9999412	500000.00	0.00
West	82° 00'	24° 20'	0.9999412	500000.00	0.00
Georgia					
East	82° 10'	30° 00'	0.9999000	500000.00	0.00
West	84° 10'	30° 00'	0.9999000	500000.00	0.00
Hawaii					
1	155° 30'	18° 50'	0.9999667	500000.00	0.00
2	156° 40'	20° 20'	0.9999667	500000.00	0.00
3	158° 00'	21° 10'	0.9999900	500000.00	0.00
4	159° 30'	21° 50'	0.9999900	500000.00	0.00
5	160° 10'	21° 40'	1.0000000	500000.00	0.00
Idaho					
East	112° 10'	41° 40'	0.9999474	500000.00	0.00
Central	114° 00'	41° 40'	0.9999474	500000.00	0.00
West	115° 45'	41° 40'	0.9999333	500000.00	0.00
Illinois					
East	88° 20'	36° 40'	0.9999750	500000.00	0.00
West	90° 10'	36° 40'	0.9999412	500000.00	0.00
Indiana					
East	85° 40'	37° 30'	0.9999667	500000.00	0.00
West	87° 05'	37° 30'	0.9999667	500000.00	0.00
Maine					
East	68° 30'	43° 50'	0.9999000	500000.00	0.00
West	70° 10'	42° 50'	0.9999667	500000.00	0.00

Transverse Mercator Co-ordinates to Latitude/Longitude

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Michigan (old)					
East	83° 40'	41° 30'	0.9999429	500000.00	0.00
Central	85° 45'	41° 30'	0.9999091	500000.00	0.00
West	88° 45'	41° 30'	0.9999091	500000.00	0.00
Mississippi					
East	88° 50'	29° 40'	0.9999600	500000.00	0.00
West	90° 20'	30° 30'	0.9999412	500000.00	0.00
Missouri					
East	90° 30'	35° 50'	0.9999333	500000.00	0.00
Central	92° 30'	35° 50'	0.9999333	500000.00	0.00
West	94° 30'	36° 10'	0.9999412	500000.00	0.00
Nevada					
East	115° 35'	34° 45'	0.9999000	500000.00	0.00
Central	116° 40'	34° 45'	0.9999000	500000.00	0.00
West	118° 35'	34° 45'	0.9999000	500000.00	0.00
New Hampshire	71° 40'	42° 30'	0.9999667	500000.00	0.00
New Jersey	74° 40'	38° 50'	0.9999750	2000000.00	0.00
New Mexico					
East	104° 20'	31° 00'	0.9999091	500000.00	0.00
Central	106° 15'	31° 00'	0.9999000	500000.00	0.00
West	107° 50'	31° 00'	0.9999167	500000.00	0.00
New York					
East	74° 20'	40° 00'	0.9999667	500000.00	0.00
Central	76° 35'	40° 00'	0.9999375	500000.00	0.00
West	78° 35'	40° 00'	0.9999375	500000.00	0.00
Rhode Island	71° 30'	41° 05'	0.9999938	500000.00	0.00
Vermont	72° 30'	42° 30'	0.9999643	500000.00	0.00

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Wyoming					
East	105° 10'	40° 40'	0.9999412	500000.00	0.00
East Central	107° 20'	40° 40'	0.9999412	500000.00	0.00
West Central	108° 45'	40° 40'	0.9999412	500000.00	0.00
West	110° 05'	40° 40'	0.9999412	500000.00	0.00

Ellipsoids

There are a range of ellipsoids in common or former use. The table below has the a and e^2 values for a number of common (and less common) ellipsoids.

Ellipsoid	a Semi-major Axis	e^2 Eccentricity
GRS80–WGS94–NAD83	6378137 m	0.006 694 38
Clarke 1866 (NAD27)	6378206.4 m	0.006 768 66
Clarke 1866 (NAD27)	20925832.2 ft	0.006 768 66
ANS (Australian)	6378160 m	0.006 694 541 855
Airy 1830	6377563.4 m	0.006 670 54
Bessel 1841	6377397.16 m	0.006 674 372
Clarke 1880	6378249.15 m	0.006 803 511
Everest 1830	6377276.35 m	0.006 637 847
Fischer 1960 (Mercury)	6378166 m	0.006 693 422
Fischer 1968	6378150 m	0.006 693 422
Hough 1956	6378270 m	0.006 722 67
International	6378388 m	0.006 722 67
Krassovsky 1940	6378245 m	0.006 693 422
South American 1960	6378160 m	0.006 694 542
GRS 1967	6378160 m	0.006 694 605
GRS 1975	6378140 m	0.006 694 385
WGS 60	6378165 m	0.006 693 422
WGS 66	6378145 m	0.006 694 542
WGS 72	6378135 m	0.006 694 317 778
WGS 84	6378137 m	0.006 694 38

Reference

SNYDER, J.P., 1987. *Map Projections—A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington: US Government Printing Office.