HP-35s Calculator Program TM 1

Convert Latitude and Longitude to Transverse Mercator Co-ordinates (UTM, SPCS, etc.)

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Line	Instruction	Display	User Instructions
T001	LBL T		↱ LBL T
T002	CLSTK		↱ CLEAR 5
T003	FS? 10		↰ FLAGS 3 .0
T004	GTO T008		
T005	SF 1		↰ FLAGS 1 1
T006	SF 10		↰ FLAGS 1 .0
T007	GTO T009		
T008	CF 1		↰ FLAGS 2 1
T009	LAT–LONG 2 TM		(Key in using EQN RCL L, RCL A, etc.)
T010	PSE		↱ PSE
T011	CL x		↱ CLEAR 1
T012	STO F		↱ STO F
T013	STO L		↱ STO L
T014	6378137		a value for ellipsoid (WGS84)
T015	STO A		↱ STO A
T016	0.00669438		e^2 value for ellipsoid (WGS84)
T017	STO E		↱ STO E
T018	0.9996		k_0 value for zone (UTM 17)
T019	STO K		↱ STO K
T020	–81		λ_0 for zone (UTM 17)
T021	STO B		↱ STO B
T022	0		ϕ_0 for zone (UTM 17)
T023	STO C		↱ STO C
T024	500000		E_0 for zone (UTM 17)
T025	STO J		↱ STO J
T026	0		N_0 for zone (UTM 17)
T027	STO I		↱ STO I
T028	CHECK–ENTER A		(Key in using EQN RCL C, RCL H, etc.)
T029	PSE		↱ PSE
T030	INPUT A		↰ INPUT A
T031	CHECK–ENTER E		(Key in using EQN RCL C, RCL H, etc.)
T032	PSE		↱ PSE
T033	INPUT E		↰ INPUT E
T034	CHECK–ENTER K		(Key in using EQN RCL C, RCL H, etc.)
T035	PSE		↱ PSE
T036	INPUT K		↰ INPUT K
T037	CHK–NTR LONG 0		(Key in using EQN RCL C, RCL H, etc.)
T038	PSE		↱ PSE

Line	Instruction
T039	INPUT B
T040	CHK–NTR LAT 0
T041	PSE
T042	INPUT C
T043	CHK–NTR N 0
T044	PSE
T045	INPUT I
T046	CHK–NTR E 0
T047	PSE
T048	INPUT J
T049	RCL B
T050	HMS→
T051	STO B
T052	RCL C
T053	HMS→
T054	STO C
T055	ENTER PT LAT
T056	PSE
T057	INPUT F
T058	ENTER PT LONG
T059	PSE
T060	INPUT L
T061	RCL F
T062	HMS→
T063	STO F
T064	RCL L
T065	HMS→
T066	STO L
****	Compute ν
T067	1
T068	RCL F
T069	SIN
T070	x^2
T071	RCL× E
T072	—
T073	√x
T074	RCL A
T075	x < > y
T076	÷
T077	STO N
****	Compute ρ
T078	1
T079	RCL— E
T080	RCL× A
T081	RCL F

Line	Instruction
T082	SIN
T083	x^2
T084	RCL× E
T085	1
T086	x < > y
T087	—
T088	1.5
T089	y^x
T090	÷
T091	STO R
****	Compute ψ
T092	RCL N
T093	RCL÷ R
T094	STO P
****	Compute t
T095	RCL F
T096	TAN
T097	STO T
****	Compute ω
T098	RCL L
T099	RCL— B
T100	→RAD
T101	STO W
****	Compute m and m_0
T102	1
T103	RCL E
T104	4
T105	÷
T106	—
T107	RCL E
T108	x^2
T109	0.046875
T110	×
T111	—
T112	RCL E
T113	3
T114	y^x
T115	0.01953125
T116	×
T117	—
T118	STO D
T119	RCL F
T120	→RAD
T121	×
T122	STO M

Line	Instruction
T123	RCL C
T124	→RAD
T125	RCL× D
T126	STO H
T127	RCL E
T128	RCL E
T129	x^2
T130	4
T131	÷
T132	+
T133	RCL E
T134	3
T135	y^x
T136	0.1171875
T137	×
T138	+
T139	0.375
T140	×
T141	STO D
T142	RCL F
T143	2
T144	×
T145	SIN
T146	×
T147	STO— M
T148	RCL C
T149	2
T150	×
T151	SIN
T152	RCL× D
T153	STO— H
T154	RCL E
T155	x^2
T156	RCL E
T157	3
T158	y^x
T159	0.75
T160	×
T161	+
T162	0.05859375
T163	×
T164	STO D
T165	RCL F
T166	4
T167	×

Latitude/Longitude to Transverse Mercator Co-ordinates

Line	Instruction
T168	SIN
T169	×
T170	STO+ M
T171	RCL C
T172	4
T173	×
T174	SIN
T175	RCL× D
T176	STO+ H
T177	RCL E
T178	3
T179	y^x
T180	35
T181	×
T182	3072
T183	÷
T184	STO D
T185	RCL F
T186	6
T187	×
T188	SIN
T189	×
T190	STO— M
T191	RCL A
T192	STO× M
T193	RCL C
T194	6
T195	×
T196	SIN
T197	RCL× D
T198	STO— H
T199	RCL A
T200	STO× H
****	Compute E
T201	RCL N
T202	RCL× W
T203	RCL F
T204	COS
T205	×
T206	STO X
T207	RCL F
T208	COS
T209	RCL× W
T210	3
T211	y^x

Line	Instruction
T212	RCL× N
T213	6
T214	÷
T215	RCL P
T216	RCL T
T217	x^2
T218	—
T219	×
T220	STO+ X
T221	1
T222	RCL T
T223	x^2
T224	6
T225	×
T226	—
T227	4
T228	×
T229	RCL P
T230	3
T231	y^x
T232	×
T233	RCL T
T234	x^2
T235	8
T236	×
T237	1
T238	+
T239	RCL P
T240	x^2
T241	×
T242	+
T243	RCL T
T244	x^2
T245	2
T246	×
T247	RCL× P
T248	—
T249	RCL T
T250	4
T251	y^x
T252	+
T253	RCL× N
T254	RCL F
T255	COS
T256	RCL× W

Line	Instruction
T257	5
T258	y^x
T259	×
T260	120
T261	÷
T262	STO+ X
T263	61
T264	RCL T
T265	x^2
T266	479
T267	×
T268	—
T269	RCL T
T270	4
T271	y^x
T272	179
T273	×
T274	+
T275	RCL T
T276	6
T277	y^x
T278	—
T279	RCL× N
T280	RCL F
T281	COS
T282	RCL× W
T283	7
T284	y^x
T285	×
T286	5040
T287	÷
T288	STO+ X
T289	RCL K
T290	STO× X
T291	RCL J
T292	STO+ X
****	Compute N
T293	RCL F
T294	COS
T295	RCL× W
T296	RCL× W
T297	2
T298	÷
T299	STO Y
T300	RCL P

Latitude/Longitude to Transverse Mercator Co-ordinates

Line	Instruction
T301	x^2
T302	4
T303	×
T304	RCL+ P
T305	RCL T
T306	x^2
T307	—
T308	RCL F
T309	COS
T310	3
T311	y^x
T312	×
T313	RCL W
T314	4
T315	y^x
T316	×
T317	24
T318	÷
T319	STO+ Y
T320	11
T321	RCL T
T322	x^2
T323	24
T324	×
T325	—
T326	8
T327	×
T328	RCL P
T329	4
T330	y^x
T331	×
T332	1
T333	RCL T
T334	x^2
T335	6
T336	×
T337	—
T338	28
T339	×
T340	RCL P
T341	3
T342	y^x
T343	×
T344	—
T345	1

Line	Instruction
T346	RCL T
T347	x^2
T348	32
T349	×
T350	—
T351	RCL× P
T352	RCL× P
T353	+
T354	RCL T
T355	x^2
T356	RCL× P
T357	2
T358	×
T359	—
T360	RCL T
T361	4
T362	y^x
T363	+
T364	RCL F
T365	COS
T366	5
T367	y^x
T368	×
T369	RCL W
T370	6
T371	y^x
T372	×
T373	720
T374	÷
T375	STO+ Y
T376	1385
T377	RCL T
T378	x^2
T379	3111
T380	×
T381	—
T382	RCL T
T383	4
T384	y^x
T385	543
T386	×
T387	+
T388	RCL T
T389	6
T390	y^x

Line	Instruction
T391	—
T392	RCL F
T393	COS
T394	7
T395	y^x
T396	×
T397	RCL W
T398	8
T399	y^x
T400	×
T401	40320
T402	÷
T403	STO+ Y
T404	RCL F
T405	SIN
T406	RCL× N
T407	STO× Y
T408	RCL M
T409	STO+ Y
T410	RCL K
T411	STO× Y
T412	RCL H
T413	RCL× K
T414	STO— Y
T415	RCL I
T416	STO+ Y
****	Compute γ
T417	0
T418	STO G
T419	RCL F
T420	SIN
T421	RCL× W
T422	STO— G
T423	RCL P
T424	x^2
T425	2
T426	×
T427	RCL— P
T428	RCL F
T429	COS
T430	x^2
T431	×
T432	RCL W
T433	3
T434	y^x

Latitude/Longitude to Transverse Mercator Co-ordinates

Line	Instruction
T435	×
T436	3
T437	÷
T438	RCL F
T439	SIN
T440	×
T441	STO— G
T442	11
T443	RCL T
T444	x^2
T445	24
T446	×
T447	—
T448	RCL P
T449	4
T450	y^x
T451	×
T452	11
T453	RCL T
T454	x^2
T455	36
T456	×
T457	—
T458	RCL P
T459	3
T460	y^x
T461	×
T462	—
T463	1
T464	RCL T
T465	x^2
T466	7
T467	×
T468	—
T469	2
T470	×
T471	RCL× P
T472	RCL× P
T473	+
T474	RCL T
T475	x^2
T476	RCL× P
T477	+
T478	RCL F
T479	COS

Line	Instruction
T480	4
T481	y^x
T482	×
T483	RCL W
T484	5
T485	y^x
T486	×
T487	15
T488	÷
T489	RCL F
T490	SIN
T491	×
T492	STO— G
T493	17
T494	RCL T
T495	x^2
T496	26
T497	×
T498	—
T499	RCL T
T500	4
T501	y^x
T502	2
T503	×
T504	+
T505	RCL F
T506	COS
T507	6
T508	y^x
T509	×
T510	RCL W
T511	7
T512	y^x
T513	×
T514	315
T515	÷
T516	RCL F
T517	SIN
T518	×
T519	STO— G
T520	RCL G
T521	→DEG
T522	→HMS
T523	STO G
****	Compute k

Line	Instruction
T524	1
T525	STO S
T526	RCL F
T527	COS
T528	RCL× W
T529	x^2
T530	2
T531	÷
T532	RCL× P
T533	STO+ S
T534	1
T535	RCL T
T536	x^2
T537	6
T538	×
T539	—
T540	RCL P
T541	3
T542	y^x
T543	×
T544	4
T545	×
T546	1
T547	RCL T
T548	x^2
T549	24
T550	×
T551	+
T552	RCL× P
T553	RCL× P
T554	+
T555	RCL T
T556	x^2
T557	RCL× P
T558	4
T559	×
T560	—
T561	RCL F
T562	COS
T563	RCL× W
T564	4
T565	y^x
T566	×
T567	24
T568	÷

Latitude/Longitude to Transverse Mercator Co-ordinates

Line	Instruction
T569	STO+ S
T570	61
T571	RCL T
T572	x^2
T573	148
T574	×
T575	—
T576	RCL T
T577	4
T578	y^x
T579	16
T580	×
T581	+
T582	RCL F
T583	COS
T584	RCL× W
T585	6
T586	y^x
T587	×
T588	720
T589	÷

Line	Instruction
T590	STO+ S
T591	RCL K
T592	STO× S
****	Show results
T593	RESULTS
T594	PSE
T595	EASTING
T596	PSE
T597	VIEW X
T598	NORTHING
T599	PSE
T600	VIEW Y
T601	GRID CONV
T602	PSE
T603	VIEW G
T604	PT SCALE FACT
T605	PSE
T606	VIEW S
T607	0
T608	STO Q
T609	NEXT PT [0—1]

Line	Instruction
T610	PSE
T611	INPUT Q
T612	RCL Q
T613	x = 0?
T614	GTO T624
T615	NEW ZONE [0—1]
T616	PSE
T617	0
T618	STO Q
T619	INPUT Q
T620	RCL Q
T621	x = 0?
T622	GTO T055
T623	GTO T028
T624	PROGRAM END
T625	PSE
T626	FS? 1
T627	CF 10
T628	RTN

Notes

(1) The program should be run in RPN mode, as results in ALG mode are unknown.

(2) Latitudes and longitudes should be entered in HP notation, i.e., DDD.MMSS. The grid convergence is displayed in HP notation.

(3) The program may be used for any Transverse Mercator projection, if the appropriate parameters are known. Similarly, any ellipsoid may be used, if its a and e^2 parameters are known. Parameters for a wide range of ellipsoids, all UTM zones and all SPCS TM zones are included at the end of this document.

(4) Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in North America. It is critical to enter the correct sign in calculator when entering values.

(5) Lines with **** are comments only, and should not be entered into the calculator. They are there to make program entry a little easier.

(6) This program is long and often appears to be a stream of meaningless commands. This means that it may be more prone to errors when being entered. It is suggested that the program be entered using the given constants, tested (and the checksum checked), and when it is satisfactory, the constants at the start of the program can be changed to those most suitable for the bulk of the expected work.

(7) The program allows the user to run additional points after each is completed, by prompting. If another point is to be processed, the user also has the option to move to a new zone and ellipsoid, otherwise the previous ellipsoid and projection parameters are

used. Respond 0 for 'NO' and 1 for 'YES' at the Q? prompt. If the user chooses to enter another point, the previous data entered is displayed at the prompts, but angular data are stored in decimal degrees. This should be re-entered in HP notation (or quickly converted with the ↱ →HMS key sequence), even if the same data is being used, because the program will convert the values to decimal degrees again, and so produce erroneous results.

(8) The program appears to work correctly, as tested. However, the grid convergence result has the opposite sign to that produced by the NGS on-line Lat/Long to SPCS conversion package at: http://www.ngs.noaa.gov/cgi-bin/spc_getpc.prl The formulae are correct in this program, and the results agree with the sign convention of Redfearn's formulae, as well as the normal usage of the grid convergence (converting between grid and true azimuths). I am not sure why the NGS program has the opposite sign, but I have asked NGS about it. Until this difference is resolved, be aware that the sign could be the opposite, and work out the correct sign from first principles.

Theory

Converting from geographical co-ordinates (latitude and longitude) to cartesian co-ordinates on a Transverse Mercator projection is a straightforward transformation, if somewhat long-winded.

Given that we have a, e^2, ϕ, k_0, λ and λ_0, we can use the following expressions for the conversion. These are Redfearn's Formulae. Note that these use an extra term in the computations of E' and N', compared to Snyder's book (1987), but this will make only a small difference in the overall values. The results will be a little different to the tabulated values for SPCS, too, owing to the limitations on the SPCS 27 computations. Remembering that the allowable distortion in the SPCS was to be no more than 1 in 10,000, it is acceptable to drop the final term in the formulae, as this doesn't degrade the formulae by anywhere near 1 in 10,000. These formulae will then agree with Snyder's formulae.

For UTM computations, you should use the full number of terms. This is because there is no 'legal' tolerance of distortion in the conversion process. UTM co-ordinates are now printed on 1:24,000 quadrangle maps, with either a grid/graticule or marginal ticks. These UTM co-ordinates are often on the NAD27 datum and need to be converted to NAD83 before they can be used. While there is a marginal note concerning the conversion of latitude and longitude from NAD27 to NAD83 on many of the more recent mapsheets, this value **does not** apply to the UTM co-ordinates (or the SPCS co-ordinates). This is because the latitude and longitude values are, in effect, figured from the origin in Kansas, while the UTM Northing co-ordinates are figured from the Equator. SPCS northings are figured from the zone origin, so will have a different shift for each zone. You should convert the co-ordinates to latitude and longitude for the appropriate system, convert these to NAD83, then convert to UTM or SPCS TM co-ordinates. An approximate set of shifts for UTM can be found in a paper by Welch, R., and Homsey, A., "Datum Shifts for UTM Co-ordinates," in the *Photogrammetric Engineering and Remote Sensing* journal, Volume 63, No. 4, pp. 371–375, published in 1997.

Latitude/Longitude to Transverse Mercator Co-ordinates

Conversion Formulae

Easting

$$
\begin{aligned}
E' = k_0 \{ & \nu \omega \cos\phi \\
& + \nu \frac{\omega^3}{6} \cos^3\phi \, (\psi - t^2) \\
& + \nu \frac{\omega^5}{120} \cos^5\phi \, [4 \psi^3 (1 - 6 t^2) + \psi^2 (1 + 8 t^2) - \psi (2 t^2) + t^4] \\
& + \nu \frac{\omega^7}{5040} \cos^7\phi \, (61 - 479 t^2 + 179 t^4 - t^6) \ \}
\end{aligned}
$$

Northing

$$
\begin{aligned}
N' = k_0 \{ m & + \nu \sin\phi \frac{\omega^2}{2} \cos\phi \\
& + \nu \sin\phi \frac{\omega^4}{24} \cos^3\phi \, (4 \psi^2 + \psi - t^2) \\
& + \nu \sin\phi \frac{\omega^6}{720} \cos^5\phi \, [8\psi^4(11-24t^2) - 28\psi^3(1-6t^2) + \psi^2(1-32t^2) - 2\psi t^2 + t^4] \\
& + \nu \sin\phi \frac{\omega^8}{40320} \cos^7\phi \, (1385 - 3111 t^2 + 543 t^4 - t^6) \ \}
\end{aligned}
$$

Grid Convergence (in radians)

$$
\begin{aligned}
\gamma = & - \sin\phi \ \omega \\
& - \sin\phi \ \frac{\omega^3}{3} \cos^2\phi \, (2 \psi^2 - \psi) \\
& - \sin\phi \ \frac{\omega^5}{15} \cos^4\phi \, [\psi^4 (11 - 24 t^2) - \psi^3 (11 - 36 t^2) + 2\psi^2 (1 - 7t^2) + \psi t^2] \\
& - \sin\phi \ \frac{\omega^7}{315} \cos^6\phi \, (17 - 26 t^2 + 2t^4)
\end{aligned}
$$

Point Scale Factor

$$
\begin{aligned}
k = k_0 \{1 & + \frac{\omega^2}{2} \cos^2\phi \ \psi \\
& + \frac{\omega^4}{24} \cos^4\phi \ [4 \psi^3 (1 - 6 t^2) + \psi^2 (1 + 24 t^2) - 4 \psi t^2] \\
& + \frac{\omega^6}{720} \cos^6\phi \ (61 - 148 t^2 + 16 t^4) \ \}
\end{aligned}
$$

where

$E' = E - {}_0E$ (E_0 is the offset of the central meridian; check the value for each zone. For UTM, E_0 = 500 000·000 meters.)

Latitude/Longitude to Transverse Mercator Co-ordinates

$N' = N - N_0$ (N_0 is the offset of the origin latitude; check the value for each zone. For UTM in the northern hemisphere, $N_0 = 0$; for UTM in the southern hemisphere, N_0 = 10 000 000·000 meters.)

ν = radius of curvature in the prime vertical at ϕ; i.e. $\nu = \dfrac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$

$$\rho = \frac{a\,(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}} = \text{radius of curvature in the meridian at } \phi$$

$\omega = \lambda - \lambda_0$

$\psi = \dfrac{\nu}{\rho}$ i.e. ratio of the radii of curvature at ϕ

$t = \tan \phi$

m = meridian distance from equator, computed using the following expression

$m = a\,(A_0\,\phi - A_2 \sin 2\phi + A_4 \sin 4\phi - A_6 \sin 6\phi)$

where ϕ is in radians and

$$A_0 = 1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256}$$

$$A_2 = \frac{3}{8}\left(e^2 + \frac{e^4}{4} + \frac{15e^6}{128}\right)$$

$$A_4 = \frac{15}{256}\left(e^4 + \frac{3e^6}{4}\right)$$

$$A_6 = \frac{35e^6}{3072}$$

With the appropriate values for ellipsoids and scale factors, these formulae will work for any Transverse Mercator projection: UTM, SPCS, AMG, MGA or whatever.

a = semi-major axis of the ellipsoid; a = 6,378,137 m for WGS84 (GRS80)

e^2 = eccentricity of the ellipsoid; e^2 = 0.006 694 3800 for WGS84.

Sample Computations

Example 1

Using the SPCS 1983 (a = 6,378,137 m, e^2 = 0.006 694 3800), the following results are obtained.

Nevada East Zone, 2701, λ_0 = –115° 35', ϕ_0 = 34° 45', k_0 = 0.999 900, E_0 = 200,000.000 m, N_0 = 8,000,000.000 m.

Latitude = 41° 25' 00" Longitude = –115° 45' 20"

Easting (E) = 185,603.123 m Northing (N) = 8,739,929.417 m

Grid Convergence (γ) = 0° 06' 50.1" Point Scale Factor (k) = 0.999 902 55

Example 2

Using the SPCS 1927 (a = 20925832.2 ft, e^2 = 0.006 768 66), the following results are obtained.

Nevada East Zone, SPCS 1927, λ_0 = –115° 35', ϕ_0 = 34° 45', k_0 = 0.999 900, E_0 = 500,000.000 ft, N_0 = 0.000 ft.

Latitude = 41° 25' 00" Longitude = –115° 45' 20"

Easting (E) = 452,764.960 ft Northing (N) = 2,427,533.222 ft

Grid Convergence (γ) = 0° 06' 50.1" Point Scale Factor (k) = 0.999 902 55

Example 3

Using the ANS ellipsoid (a = 6,378,160 m, e^2 = 0.006 694 541 855), the following results are obtained.

AMG Zone 54, λ_0 = +141° 00', ϕ_0 = 0° 00', k_0 = 0.999 600,
E_0 = 500,000.000 m, N_0 = 10,000,000.000 m.

Latitude = –37° 39' 15".5571 Longitude = +143° 55' 30".6330

Easting (E) = 758,053.090 m Northing (N) = 5,828,496.973 m

Grid Convergence (γ) = +1° 47' 16.67" Point Scale Factor (k) = 1.000 420 30

These results agree with those computed in the AGD Technical Manual, 1986.

Running the Program

Press XEQ T , then the ENTER key, to start the program. The calculator briefly displays LAT—LONG 2 TM, then briefly shows CHECK—ENTER A. This is “Point A,” discussed below. The program then stops and displays the prompt for entering the semi-major axis value, while displaying the current default value:

A?
6,378,137.0000 (This is for GRS80/WGS84/NAD83)

If you are happy with this value for the semi-major axis of the ellipsoid, press R/S to continue. Otherwise. Key in a different value (for a different ellipsoid) and press R/S to continue.

The calculator briefly displays CHECK—ENTER E. The program then stops and displays the prompt for entering the eccentricity of the ellipsoid, e:

E?
0.00669438 (This is for GRS80/WGS84/NAD83)

If this value for the eccentricity is correct, press R/S to continue. Otherwise, key in a different value (for a different ellipsoid) and press R/S to continue.

The calculator briefly displays CHECK—ENTER K. The program then stops and displays the prompt for entering the scale factor at the central meridian (λ_0), which is k_0:

K?
0.9996000 (This is for UTM)

If this value for the scale factor is satisfactory, press R/S to continue. If you want to change it, such as for an SPCS zone, key in the correct value and press R/S. In this case, key in 0.9999 for Nevada East (2701).

The calculator briefly displays CHK—NTR LONG 0. The program then stops and displays the prompt for entering the longitude of the central meridian of the projection, λ_0. Note that in the western hemisphere, this will be a negative value, and should be in HP notation (DDD.MMSS).

B?
—81.000000 (This is for UTM Zone 17)

If this is the correct central meridian, press R/S to continue, if this is not correct, key in the correct value, in HP notation, then press R/S to continue. In this case, key in –115.35 for Nevada East (2701).

The calculator briefly displays CHK—NTR LAT 0. The program then stops and displays the prompt for entering the latitude of the Northing co-ordinate origin, ϕ_0. For UTM, this is 0.000 (the equator), while for SPCS Zones, it is usually a latitude well south of the zone. The value should be entered in HP notation.

C?
0.000000 (This is for UTM)

If this is the correct latitude base, press R/S to continue. If you want a different value, key in that value and press R/S to continue. In this case, key in 34.45 and press R/S to continue.

The calculator briefly displays CHK—NTR N0. The program then stops and prompts for entry of the false northing value, or the northing offset. This is the value of the northing co-ordinate at ϕ_0, λ_0. For UTM in the northern hemisphere, this is 0.0000, while its value varies for different SPCS zones.

I?
0.0000 (This is for UTM)

If this is the correct value, press R/S to continue. If a different value is desired, key in the value and press R/S. In this case, key in 8,000,000.0 and press R/S. This is the N_0 value for Nevada East 2701.

The calculator briefly displays CHK—NTR E0. The program then stops and prompts for the false easting value, or the easting offset. This is the value of the easting at the central meridian (λ_0), denoted E_0.

J?
500,000.0000 (This is for UTM)

If this is the correct value, press R/S to continue. If a different value is required, key in the value and press R/S. In this case, key in 200,000.000 and press R/S. This is the E_0 value for Nevada East 2701.

This is "Point B," discussed below. The calculator briefly displays ENTER PT LAT. The program stops and displays the prompt for entering the latitude of the point to be converted. This should be in HP notation.

F?
0.0000

Key in the latitude of the point in HP notation and press R/S to continue. In this case, key in 41.25 and press R/S.

Latitude/Longitude to Transverse Mercator Co-ordinates

The calculator briefly displays ENTER PT LONG. The program then stops and displays the prompt for entering the longitude of the point to be converted. This should be in HP notation.

L?
0.0000

Key in the longitude of the point in HP notation and press R/S to continue. In this case, key in —115.452 and press R/S.

The program displays RUNNING for a short while, then displays RESULTS briefly, followed by EASTING briefly. The program then stops and displays the easting value of the point. In this case, the calculator displays:

X=
185,603.1225

This is the easting of the point, in this case in meters. Press R/S to continue. The calculator briefly displays NORTHING, then stops and displays the northing value of the point. In this case, the calculator displays:

Y=
8,739,929.4173

This is the northing of the point, in this case in meters. Press R/S to continue. The calculator briefly displays GRID CONV, then stops and displays the grid convergence value in HP notation. In this case, the calculator displays:

G=
0.0650149

This is the grid convergence in HP notation, and is 0° 06' 50".149 in more conventional notation. Press R/S to continue. The calculator briefly displays PT SCALE FACT, then stops and displays the point scale factor of the point on the Transverse Mercator projection. In this case, the calculator displays:

S=
0.999902550

This is the point scale factor. Press R/S to continue.

You now have the choice of running one or more additional points. The calculator briefly displays NEXT PT [0—1], then stops and displays the prompt for answering questions:

Q?
0.0000

If you want to quit the program, just press R/S. If you want to enter more points, key in 1 and press R/S. In this case, the calculator then prompts to see if you want to use the same parameters. The calculator briefly displays NEW ZONE [0—1], then stops at the question prompt:

Q?
0.0000

If you want to go to a new zone, key in 1 and press R/S, and the calculator will take you to the point where you can change any of the values (Point A above), starting with the ellipsoid parameters. If you want to work in the same zone already entered, just press R/S, and the program will take you to “Point B” and prompt for the latitude of the point to be converted, and continue from there. You can go around the program as many times as necessary.

When you choose to end the program, the calculator briefly displays PROGRAM END and then comes to an end, returning to the point from which it was called, or to normal operations, and resetting Flag 10.

Storage Registers Used

A	Semi-major axis of the ellipsoid being used, a
B	λ_0, the central meridian of the projection
C	ϕ_0, the origin latitude for the co-ordinates
D	temporary storage
E	Eccentricity of the ellipsoid, e^2
F	λ, latitude of the point to be converted
G	γ, the grid convergence of the point being converted
H	meridian distance of the origin latitude, ϕ_0
I	N_0, the offset for the northings (the northing at ϕ_0, λ_0)
J	E_0, the offset for the eastings (the easting at the central meridian, λ_0)
K	k_0, the scale factor at the central meridian, λ_0
L	λ, longitude of the point to be converted
M	m, meridian distance of the point to be converted
N	ν
P	$\psi = \dfrac{\nu}{\rho}$
Q	used for getting responses to questions about running more points
R	ρ
S	k, point scale factor at the point being converted
T	$\tan \phi$
W	$\omega = \lambda - \lambda_0$
X	Easting co-ordinate of converted point
Y	Northing co-ordinate of converted point

Statistical Registers: not used

Labels Used

Label **T** Length = 2332 Checksum = AAF9

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computations to check proper operation after entry.

Latitude/Longitude to Transverse Mercator Co-ordinates

Flags Used

Flags 1 and 10 are used by this program. Flag 10 is set for this program, so that equations can be shown as prompts. Flag 1 is used to record the setting of Flag 10 before the program begins. At the end of the program, Flag 10 is reset to its original value, based on the value in Flag 1.

Parameters for the Computations

Universal Transverse Mercator (UTM)

For UTM, the ϕ_0 value is 0° (the equator) for both northern and southern hemispheres. The λ_0 values are given for each zone in the table below.

Zone	Central Meridian, λ_0	Zone	Central Meridian, λ_0
1	177° W	31	3°E
2	171° W	32	9°E
3	165° W	33	15°E
4	159° W	34	21°E
5	153° W	35	27°E
6	147° W	36	33°E
7	141° W	37	39°E
8	135° W	38	45°E
9	129° W	39	51°E
10	123° W	40	57°E
11	117° W	41	63°E
12	111° W	42	69°E
13	105° W	43	75°E
14	99° W	44	81°E
15	93° W	45	87°E
16	87° W	46	93°E
17	81° W	47	99°E
18	75° W	48	105°E
19	69° W	49	111°E
20	63° W	50	117°E
21	57° W	51	123°E
22	51° W	52	129°E
23	45° W	53	135°E
24	39° W	54	141°E
25	33° W	55	147°E
26	27° W	56	153°E
27	21° W	57	159°E
28	15° W	58	165°E
29	9° W	59	171°E
30	3° W	60	177°E

The E_0 value for all zones is 500,000.000 meters. The N_0 value for the northern hemisphere is 0.000 meters. The N_0 value for the southern hemisphere is 10,000,000.000 meters.

State Plane Co-ordinate System (SPCS) 1983

Several US states use the Transverse Mercator projection for SPCS 1983. The various parameters for each zone in the 1983 system are given in the table below. Use these parameters with the program, together with the GRS80/WGS84/NAD83 ellipsoid parameters, in meters.

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
Alabama					
East	85° 50'	30° 30'	0.9999600	200000.00	0.00
West	87° 30'	30° 00'	0.9999333	600000.00	0.00
Alaska					
2	142° 00'	54° 00'	0.9999000	500000.00	0.00
3	146° 00'	54° 00'	0.9999000	500000.00	0.00
4	150° 00'	54° 00'	0.9999000	500000.00	0.00
5	154° 00'	54° 00'	0.9999000	500000.00	0.00
6	185° 00'	54° 00'	0.9999000	500000.00	0.00
7	162° 00'	54° 00'	0.9999000	500000.00	0.00
8	166° 00'	54° 00'	0.9999000	500000.00	0.00
9	170° 00'	54° 00'	0.9999000	500000.00	0.00
Arizona					
East	110° 10'	31° 00'	0.9999000	213360.00	0.00
Central	111° 55'	31° 00'	0.9999000	213360.00	0.00
West	113° 45'	31° 00'	0.9999333	213360.00	0.00
Delaware	72° 25'	38° 00'	0.9999950	200000.00	0.00
Florida					
East	81° 00'	24° 20'	0.9999412	200000.00	0.00
West	82° 00'	24° 20'	0.9999412	200000.00	0.00
Georgia					
East	82° 10'	30° 00'	0.9999000	200000.00	0.00
West	84° 10'	30° 00'	0.9999000	700000.00	0.00
Hawaii					
1	155° 30'	18° 50'	0.9999667	500000.00	0.00
2	156° 40'	20° 20'	0.9999667	500000.00	0.00
3	158° 00'	21° 10'	0.9999900	500000.00	0.00
4	159° 30'	21° 50'	0.9999900	500000.00	0.00
5	160° 10'	21° 40'	1.0000000	500000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
Idaho					
East	112° 10'	41° 40'	0.9999474	200000.00	0.00
Central	114° 00'	41° 40'	0.9999474	500000.00	0.00
Illinois					
East	88° 20'	36° 40'	0.9999750	300000.00	0.00
West	90° 10'	36° 40'	0.9999412	700000.00	0.00
Indiana					
East	85° 40'	37° 30'	0.9999667	100000.00	250000.00
West	87° 05'	37° 30'	0.9999667	900000.00	250000.00
Maine					
East	68° 30'	43° 40'	0.9999000	300000.00	0.00
West	70° 10'	42° 50'	0.9999667	900000.00	0.00
Mississippi					
East	88° 50'	29° 30'	0.9999500	300000.00	0.00
West	90° 20'	29° 30'	0.9999500	700000.00	0.00
Missouri					
East	90° 30'	35° 50'	0.9999333	250000.00	0.00
Central	92° 30'	35° 50'	0.9999333	500000.00	0.00
West	94° 30'	36° 10'	0.9999412	850000.00	0.00
Nevada					
East	115° 35'	34° 45'	0.9999000	200000.00	8000000.00
Central	116° 40'	34° 45'	0.9999000	500000.00	6000000.00
West	118° 35'	34° 45'	0.9999000	800000.00	4000000.00
New Hampshire	71° 40'	42° 30'	0.9999667	300000.00	0.00
New Jersey	74° 30'	38° 50'	0.9999000	150000.00	0.00
New Mexico					
East	104° 20'	31° 00'	0.9999091	165000.00	0.00
Central	106° 15'	31° 00'	0.9999000	500000.00	0.00
West	107° 50'	31° 00'	0.9999167	830000.00	0.00

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
New York					
East	74° 30'	40° 00'	0.9999000	150000.00	0.00
Central	76° 35'	40° 00'	0.9999375	250000.00	0.00
West	78° 35'	40° 00'	0.9999375	350000.00	0.00
Rhode Island	71° 30'	41° 05'	0.9999938	100000.00	0.00
Vermont	72° 30'	42° 30'	0.9999643	500000.00	0.00
Wyoming					
East	105° 10'	40° 30'	0.9999375	200000.00	0.00
East Central	107° 20'	40° 30'	0.9999375	400000.00	100000.00
West Central	108° 45'	40° 30'	0.9999375	600000.00	0.00
West	110° 05'	40° 30'	0.9999375	800000.00	100000.00

State Plane Co-ordinate System (SPCS) 1927

Several US states used the Transverse Mercator projection for SPCS 1927. The various parameters for each zone in the 1927 system are given in the table below. Use these parameters with the program, together with the Clarke 1866 ellipsoid in feet.

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Alabama					
East	85° 50'	30° 30'	0.9999600	500000.00	0.00
West	87° 30'	30° 00'	0.9999333	500000.00	0.00
Alaska					
2	142° 00'	54° 00'	0.9999000	500000.00	0.00
3	146° 00'	54° 00'	0.9999000	500000.00	0.00
4	150° 00'	54° 00'	0.9999000	500000.00	0.00
5	154° 00'	54° 00'	0.9999000	500000.00	0.00
6	185° 00'	54° 00'	0.9999000	500000.00	0.00
7	162° 00'	54° 00'	0.9999000	700000.00	0.00
8	166° 00'	54° 00'	0.9999000	500000.00	0.00
9	170° 00'	54° 00'	0.9999000	600000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Arizona					
East	110° 10'	31° 00'	0.9999000	500000.00	0.00
Central	111° 55'	31° 00'	0.9999000	500000.00	0.00
West	113° 45'	31° 00'	0.9999333	500000.00	0.00
Delaware	72° 25'	38° 00'	0.9999950	500000.00	0.00
Florida					
East	81° 00'	24° 20'	0.9999412	500000.00	0.00
West	82° 00'	24° 20'	0.9999412	500000.00	0.00
Georgia					
East	82° 10'	30° 00'	0.9999000	500000.00	0.00
West	84° 10'	30° 00'	0.9999000	500000.00	0.00
Hawaii					
1	155° 30'	18° 50'	0.9999667	500000.00	0.00
2	156° 40'	20° 20'	0.9999667	500000.00	0.00
3	158° 00'	21° 10'	0.9999900	500000.00	0.00
4	159° 30'	21° 50'	0.9999900	500000.00	0.00
5	160° 10'	21° 40'	1.0000000	500000.00	0.00
Idaho					
East	112° 10'	41° 40'	0.9999474	500000.00	0.00
Central	114° 00'	41° 40'	0.9999474	500000.00	0.00
West	115° 45'	41° 40'	0.9999333	500000.00	0.00
Illinois					
East	88° 20'	36° 40'	0.9999750	500000.00	0.00
West	90° 10'	36° 40'	0.9999412	500000.00	0.00
Indiana					
East	85° 40'	37° 30'	0.9999667	500000.00	0.00
West	87° 05'	37° 30'	0.9999667	500000.00	0.00
Maine					
East	68° 30'	43° 50'	0.9999000	500000.00	0.00
West	70° 10'	42° 50'	0.9999667	500000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Michigan (old)					
East	83° 40'	41° 30'	0.9999429	500000.00	0.00
Central	85° 45'	41° 30'	0.9999091	500000.00	0.00
West	88° 45'	41° 30'	0.9999091	500000.00	0.00
Mississippi					
East	88° 50'	29° 40'	0.9999600	500000.00	0.00
West	90° 20'	30° 30'	0.9999412	500000.00	0.00
Missouri					
East	90° 30'	35° 50'	0.9999333	500000.00	0.00
Central	92° 30'	35° 50'	0.9999333	500000.00	0.00
West	94° 30'	36° 10'	0.9999412	500000.00	0.00
Nevada					
East	115° 35'	34° 45'	0.9999000	500000.00	0.00
Central	116° 40'	34° 45'	0.9999000	500000.00	0.00
West	118° 35'	34° 45'	0.9999000	500000.00	0.00
New Hampshire	71° 40'	42° 30'	0.9999667	500000.00	0.00
New Jersey	74° 40'	38° 50'	0.9999750	2000000.00	0.00
New Mexico					
East	104° 20'	31° 00'	0.9999091	500000.00	0.00
Central	106° 15'	31° 00'	0.9999000	500000.00	0.00
West	107° 50'	31° 00'	0.9999167	500000.00	0.00
New York					
East	74° 20'	40° 00'	0.9999667	500000.00	0.00
Central	76° 35'	40° 00'	0.9999375	500000.00	0.00
West	78° 35'	40° 00'	0.9999375	500000.00	0.00
Rhode Island	71° 30'	41° 05'	0.9999938	500000.00	0.00
Vermont	72° 30'	42° 30'	0.9999643	500000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Wyoming					
East	105° 10'	40° 40'	0.9999412	500000.00	0.00
East Central	107° 20'	40° 40'	0.9999412	500000.00	0.00
West Central	108° 45'	40° 40'	0.9999412	500000.00	0.00
West	110° 05'	40° 40'	0.9999412	500000.00	0.00

Ellipsoids

There are a range of ellipsoids in common or former use. The table below has the a and e^2 values for a number of common (and less common) ellipsoids.

Ellipsoid	a Semi-major Axis	e^2 Eccentricity
GRS80–WGS94–NAD83	6378137 m	0.006 694 38
Clarke 1866 (NAD27)	6378206.4 m	0.006 768 66
Clarke 1866 (NAD27)	20925832.2 ft	0.006 768 66
ANS (Australian)	6378160 m	0.006 694 541 855
Airy 1830	6377563.4 m	0.006 670 54
Bessel 1841	6377397.16 m	0.006 674 372
Clarke 1880	6378249.15 m	0.006 803 511
Everest 1830	6377276.35 m	0.006 637 847
Fischer 1960 (Mercury)	6378166 m	0.006 693 422
Fischer 1968	6378150 m	0.006 693 422
Hough 1956	6378270 m	0.006 722 67
International	6378388 m	0.006 722 67
Krassovsky 1940	6378245 m	0.006 693 422
South American 1960	6378160 m	0.006 694 542
GRS 1967	6378160 m	0.006 694 605
GRS 1975	6378140 m	0.006 694 385
WGS 60	6378165 m	0.006 693 422
WGS 66	6378145 m	0.006 694 542
WGS 72	6378135 m	0.006 694 317 778
WGS 84	6378137 m	0.006 694 38

Reference

SNYDER, J.P., 1987. *Map Projections—A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington: US Government Printing Office.