HP-35s Calculator Program — Curves 1A

Solve the Parameters of a Circular Horizontal Curve, given any two Parameters

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Date: April, 2008. **Version**: 1.1 **Mnemonic**: **S** for Curve **S**olution.

Line	Instruction	Display	User Instructions
S001	LBL S		LBL S
S002	CLSTK		CLEAR 5
S003	FS? 10		FLAGS 3 .0
S004	GTO S008		
S005	SF 1		FLAGS 1 1
S006	SF 10		FLAGS 1 .0
S007	GTO S009		
S008	CF 1		FLAGS 2 1
S009	SOLVE HZ CURVE		(Key in using EQN RCL S, RCL O, etc.)
S010	PSE		PSE
S011	CLx		CLEAR 1
S012	STO C		STO C
S013	STO R		STO R
S014	STO Q		STO Q
S015	STO T		STO T
S016	STO A		STO A
S017	CLΣ		CLEAR 4
S018	CHORD LENGTH		(Key in using EQN RCL C, RCL H, etc.)
S019	PSE		PSE
S020	INPUT C	C?	INPUT C
S021	x ≠ 0 ?		x?0 1
S022	Σ+		
S023	RADIUS		(Key in using EQN RCL R, RCL A, etc.)
S024	PSE		PSE
S025	INPUT R	R?	INPUT R
S026	x ≠ 0 ?		x?0 1
S027	Σ+		
S028	DEFLECTION θ		(Key in using EQN RCL D, RCL E, etc.)
S029	PSE		PSE
S030	INPUT Q	Q?	INPUT Q
S031	RCL Q		
S032	HMS→		HMS→
S033	STO Q		STO Q
S034	x ≠ 0 ?		x?0 1
S035	Σ+		
S036	TANGENT LENGTH		(Key in using EQN RCL T, RCL A, etc.)
S037	PSE		PSE
S038	INPUT T	T?	INPUT T
S039	x ≠ 0 ?		x?0 1

Line	Instruction
S040	Σ+
S041	ARC LENGTH
S042	PSE
S043	INPUT A
S044	x ≠ 0 ?
S045	Σ+
S046	2
S047	n *
S048	x < y?
S049	GTO S103
S050	x = y?
S051	GTO S057
S052	ONLY 2 INPUTS
S053	PSE
S054	SET 1 TO 0!
S055	PSE
S056	GTO S011
****	Start of "switch"
S057	RCL C
S058	x = 0?
S059	GTO S073
S060	RCL R
S061	x ≠ 0?
S062	GTO S108
S063	RCL Q
S064	x ≠ 0?
S065	GTO S112
S066	RCL T
S067	x ≠ 0?
S068	GTO S116
S069	RCL A
S070	x ≠ 0?
S071	GTO S140
S072	GTO S103
S073	RCL R
S074	x = 0?
S075	GTO S086
S076	RCL Q
S077	x ≠ 0?
S078	GTO S120
S079	RCL T
S080	x ≠ 0?

Line	Instruction
S081	GTO S124
S082	RCL A
S083	x ≠ 0?
S084	GTO S128
S085	GTO S103
S086	RCL Q
S087	x = 0?
S088	GTO S096
S089	RCL T
S090	x ≠ 0?
S091	GTO S132
S092	RCL A
S093	x ≠ 0?
S094	GTO S136
S095	GTO S103
S096	RCL T
S097	x = 0?
S098	GTO S103
S099	RCL A
S100	x = 0?
S101	GTO S103
S102	GTO S144
****	Error message
S103	NOT ENOUGH
S104	PSE
S105	DATA! RE–ENTER
S106	PSE
S107	GTO S011
****	C, R, known
S108	XEQ S185
S109	XEQ S255
S110	XEQ S215
S111	GTO S147
****	C, Q, known
S112	XEQ S221
S113	XEQ S255
S114	XEQ S215
S115	GTO S147
****	C, T, known
S116	XEQ S206
S117	XEQ S221
S118	XEQ S215

Line	Instruction
S119	GTO S147
****	R, Q, known
S120	XEQ S246
S121	XEQ S255
S122	XEQ S215
S123	GTO S147
****	R, T, known
S124	XEQ S199
S125	XEQ S246
S126	XEQ S215
S127	GTO S147
****	R, A, known
S128	XEQ S194
S129	XEQ S246
S130	XEQ S255
S131	GTO S147
****	Q, T, known
S132	XEQ S237
S133	XEQ S221
S134	XEQ S215
S135	GTO S147
****	Q, A, known
S136	XEQ S231
S137	XEQ S246
S138	XEQ S255
S139	GTO S147
****	C, A, known
S140	XEQ S265
S141	XEQ S221
S142	XEQ S255
S143	GTO S147
****	T, A, known
S144	XEQ S284
S145	XEQ S231
S146	XEQ S246
****	Seg area & show
S147	RCL Q
S148	→RAD
S149	RCL Q
S150	SIN
S151	—
S152	RCL R

* This is the statistical count, retrieved using SUMS n.

**** These lines are simply comments in the code. You don't key it into the calculator!

Line	Instruction
S153	x^2
S154	×
S155	2
S156	÷
S157	STO B
S158	SOLUTION
S159	PSE
S160	CHORD
S161	PSE
S162	VIEW C
S163	RADIUS
S164	PSE
S165	VIEW R
S166	TANGENT
S167	PSE
S168	VIEW T
S169	ARC LENGTH
S170	PSE
S171	VIEW A
S172	RCL Q
S173	→HMS
S174	STO Q
S175	DEFLECTION θ
S176	PSE
S177	VIEW Q
S178	RCL Q
S179	HMS→
S180	STO Q
S181	SEGMENT AREA
S182	PSE
S183	VIEW B
S184	GTO S305
****	Calculate Q – 1
S185	RCL C
S186	2
S187	÷
S188	RCL÷ R
S189	ASIN
S190	2
S191	×
S192	STO Q
S193	RTN
****	Calculate Q – 2
S194	RCL A
S195	RCL÷ R

Line	Instruction
S196	→DEG
S197	STO Q
S198	RTN
****	Calculate Q – 3
S199	RCL T
S200	RCL÷ R
S201	ATAN
S202	2
S203	×
S204	STO Q
S205	RTN
****	Calculate Q – 4
S206	RCL C
S207	2
S208	÷
S209	RCL÷ T
S210	ACOS
S211	2
S212	×
S213	STO Q
S214	RTN
****	Calculate A
S215	RCL R
S216	RCL Q
S217	→RAD
S218	×
S219	STO A
S220	RTN
****	Calculate R – 1
S221	RCL C
S222	2
S223	÷
S224	RCL Q
S225	2
S226	÷
S227	SIN
S228	÷
S229	STO R
S230	RTN
****	Calculate R – 2
S231	RCL A
S232	RCL Q
S233	→RAD
S234	÷
S235	STO R

Line	Instruction
S236	RTN
****	Calculate C – 1
S237	2
S238	RCL× T
S239	RCL Q
S240	2
S241	÷
S242	COS
S243	×
S244	STO C
S245	RTN
****	Calculate C – 2
S246	2
S247	RCL× R
S248	RCL Q
S249	2
S250	÷
S251	SIN
S252	×
S253	STO C
S254	RTN
****	Calculate T
S255	RCL C
S256	2
S257	÷
S258	RCL Q
S259	2
S260	÷
S261	COS
S262	÷
S263	STO T
S264	RTN
****	Calculate Q (AC)
S265	0
S266	STO U
S267	RCL A
S268	RCL÷ C
S269	1
S270	—
S271	0.06
S272	÷
S273	√x
S274	→DEG
S275	STO Q
S276	FN= U

Line	Instruction
S277	SOLVE Q
S278	RTN
S279	CANNOT SOLVE
S280	PSE
S281	WITH THESE DATA
S282	PSE
S283	GTO S305
****	Calculate Q (AT)
S284	1
S285	STO U
S286	RCL T
S287	RCL÷ A
S288	0.5
S289	—
S290	7
S291	×
S292	RCL A
S293	RCL÷ T
S294	x^2
S295	÷
S296	→DEG
S297	STO Q
S298	FN= U
S299	SOLVE Q
S300	RTN
S301	CANNOT SOLVE
S301	PSE
S303	WITH THESE DATA
S304	PSE
****	End part
S305	FS? 1
S306	CF 10
S307	STOP
S308	RTN

Line	Instruction
U001	LBL U
U002	RCL U
U003	x = 0?
U004	GTO U017
U005	RCL Q
U006	2
U007	÷
U008	TAN
U009	RCL Q
U010	→RAD
U011	÷
U012	RCL T
U013	RCL÷ A
U014	—
U015	→DEG
U016	RTN
U017	RCL Q
U018	2
U019	÷
U020	SIN
U021	2
U022	×
U023	RCL Q
U024	→RAD
U025	÷
U026	RCL C
U027	RCL÷ A
U028	—
U029	→DEG
U030	RTN

Notes

1. The **** lines are comments and are not to be entered into the calculator. They are there to make it easier to work through entering a long program.

2. Be very careful when entering the line numbers in the various XEQ and GTO statements.

3. Angles are entered and displayed in HP notation (DDD.MMSSss).

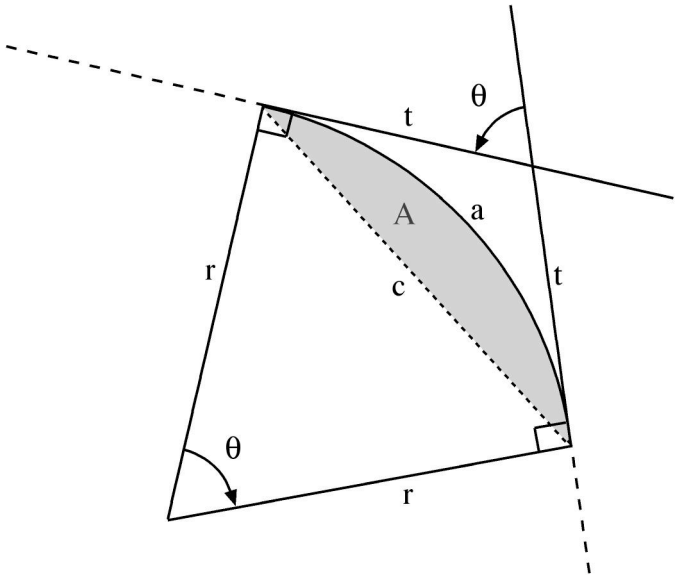
4. The program will not work for parameters that are the result of a deflection angle greater than or equal to 180°. This produces a "division by zero" error. Similarly, "impossible figures" will not produce correct results.

5. Some pairs of parameters have considerable sensitivity to small variations in their values. Therefore, consider doing a little sensitivity analysis (e.g., re-do the calculation with the parameters changed by an amount about equal to the expected error in them) to see what a reasonable precision of the result might be.

6. This program is designed to work with exactly two parameters. If you have more or fewer, the program will demand that you use only two. Choose the two most suitable parameters and ignore the others, using them as a check on the values produced. The program cannot do an adjustment based on redundant data.

Theory

The theory of solving the parameters of a horizontal circular curve is fairly straightforward. Given a curve as shown in the figure below, the various parameters are related through the following equations. Therefore, given any two parameters, it is possible to solve for all the others.



In this situation, θ is the deflection angle, or angle at the center of the arc; c is the chord length; r is the radius; a is the length of the arc of the curve; t is the length of the tangent, from the tangent point to the intersection point; and A is the area of the segment between the arc and the chord (shown with gray shading).

The perpendicular bisector of the chord also bisects the angle at the center of the curve (θ), dividing the quadrilateral into two congruent right triangles, and the isosceles triangle formed by the radii and the chord into two other congruent right triangles. Solving these triangles in various ways allows any two parameters to solve most of the other parameters. The formulae used are as follows:

$$\theta = 2\arcsin\left(\frac{c/2}{r}\right) = 2\arccos\left(\frac{c/2}{t}\right) = 2\arctan\left(\frac{t}{r}\right)$$

$$r = \frac{c/2}{\tan\left(\theta/2\right)} = \frac{a}{\theta}$$

$$c = 2r\sin\left(\frac{\theta}{2}\right) = 2t\cos\left(\frac{\theta}{2}\right)$$

$$t = \frac{c/2}{\cos\left(\theta/2\right)}$$

$$a = r\theta$$

$$A = \frac{1}{2}r^2\left(\theta - \sin\theta\right)$$

When θ is used by itself, it usually denotes its use as a radian value.

In the event that the chord and arc, or the tangent and arc, are the only values known, the solution is not direct. Instead, the following equations are set up (in the subprogram with label U), for each case:

$$\frac{2\sin\left(\frac{\theta}{2}\right)}{\theta} - \frac{c}{a} = 0 \quad \text{and} \quad \frac{\tan\left(\frac{\theta}{2}\right)}{\theta} - \frac{t}{a} = 0 \quad \text{respectively.}$$

These are solved for θ using the HP Solve capability in the calculator, after a starting estimate for θ is calculated.

Running the Program

Key in XEQ S then press the Enter key. The program starts and displays:

SOLVE HZ CURVE

then prompts for the chord length, displaying:

CHORD LENGTH

then stops while displaying:

C?
0.0000

If the length of the chord is known, key it in and press R/S. If it is not known, leave the value at zero and press R/S. The calculator then displays:

RADIUS

then stops while displaying:

R?
0.0000

If the radius is known, key it in, then press R/S. If it is not known, leave the value at zero and press R/S. The calculator then displays:

DEFLECTION θ

then stops while displaying:

Q?
0.0000

If the value of the deflection angle is known, key it in here in DDD.MMSSss format (HP notation), then press R/S. if the deflection angle is not known, leave the value at zero and press R/S. The calculator then displays:

TANGENT LENGTH

then stops while displaying:

T?
0.0000

If the length of the tangent is known, key it in here and press R/S. If it is not known, leave the value at zero and press R/S. The calculator then displays:

ARC LENGTH

then stops while displaying:

A?
0.0000

If the arc length is known, key it in here and press R/S. If it is not known, leave the alue at zero and press R/S.

If you have entered fewer than two parameter values, i.e., there are fewer than two non-zero values, the calculator briefly displays:

NOT ENOUGH then, briefly:
DATA! RE-ENTER

and returns to prompting for the chord length, as above. You then are prompted for all the other possible data values, in turn, as shown above.

If you have entered more than two parameter values, the calculator briefly displays:

ONLY 2 INPUTS then, briefly:
SET 1 to 0!

and returns to prompting for the chord length, as above. You then are prompted for all the other possible data values, in turn, as shown above.

If you have entered exactly two parameter values, the calculator displays:

RUNNING then, briefly:

SOLUTION then, briefly:

CHORD

then stops and shows the chord value, e.g.:

C=
258.8190

Press R/S. The calculator briefly displays:

RADIUS

then stops and shows the radius value, e.g.:

R=
500.0000

Press R/S. The calculator briefly displays:

TANGENT

then stops and shows the tangent length, e.g.:

T=
133.9750

Press R/S/. The calculator briefly displays:

ARC LENGTH

then stops and shows the arc length, e.g.:

A=
261.7990

Press R/S. The calculator briefly displays:

DEFLECTION θ

then stops and shows the deflection angle, q, in HP notation (DDD.MMSSss format), e.g.:

Q=
30.595900

Press R/S. The calculator briefly displays:

SEGMENT AREA

then stops and shows the area of the segment between the chord and the arc, e.g.:

B=
34,199.8470

Press R/S. The program resets flag 10 to its original value, then stops and returns to normal calculator operation.

In the event that the parameters entered were the chord and arc lengths, or the tangent and arc lengths, the solution will take a little longer, and the calculator will display:

SOLVING

for a short time, while the HP Solve process is being done. As this is the first step in both cases, it is followed by the calculator displaying:

RUNNING

before moving to display the solution.

Sample Computations

	1	2	3
Radius	500.000	500.000	500.000
Deflection Angle	30.000	45.000	60.000
Chord Length	258.819	382.683	500.000
Tangent Length	133.975	207.107	288.675
Arc Length	261.799	392.699	523.599
Segment Area	2,949.847	9,786.423	22,646.518

Entering various combinations of any two values for any one solution should give the other parameter values. However, there may be some sensitivity when various input parameters are used, so that there will be some small variation in the output parameters in some cases, In

particular, the area may change by small amounts, and solutions that start with the arc length are sometimes particularly sensitive.

	4	**5**	**6**
Radius	500.000	500.000	500.000
Deflection Angle	90.000	120.000	150.000
Chord Length	707.107	866.025	965.926
Tangent Length	500.000	866.025	1,866.025
Arc Length	785.398	1,047.198	1,308.997
Segment Area	71,349.541	153,546.212	264,749.235

	7	**8**	**9**
Radius	500.000	500.000	250.000
Deflection Angle	170.000	105.000	109.4522
Chord Length	996.195	793.353	408.965
Tangent Length	5,715.026	651.613	355.425
Arc Length	1,483.530	916.298	478.901
Segment Area	349,176.444	108,333.736	30,452.048

Storage Registers Used

A Arc length (a).

B Segment area (A).

C Chord length (c).

Q Deflection angle (θ).

R Radius (r).

T Tangent length (t).

U Selector for TA or CA solutions in HP Solve.

Statistical Registers: Used to count the number of parameters entered, only the count (n) is used.

Labels Used

Label **S** Length = 1180 Checksum = F62D

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Routines Called

The program labeled U looks at the value in the storage register U, and uses this to decide whether a TA (tangent and arc lengths) or CA (chord and arc length) solution is needed, then jumps to the part of the code that implements the equation to be solved. The HP Solve package in the calculator uses the code under label U as the basis for solving for the deflection angle (θ, stored in Q the program), when called from the main program (under label S). The subprogram under label U accesses the storage locations A, Q, and C or T, as needed, for the solution.

Label **U** Length = 93 Checksum = 894F

Flags Used

Flags 1 and 10 are used by this program. Flag 10 is set for this program, so that equations can be shown as prompts. Flag 1 is used to record the setting of Flag 10 before the program begins. At the end of the program, Flag 10 is reset to its original value, based on the value in Flag 1.

Special Thanks

The original version of the program had an error in the formula for the area of the segment, which was also in the code, and hence in the examples. There were some other typos in the examples, as well. Asa Ramsay was kind enough to bring these to my attention, allowing me to fix them quickly and post a revised program.