Convert XYZ Geocentric Co-ordinates on any ellipsoid to Latitude, Longitude and Height

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Line	Instruction	Display	User Instructions
Y0001	LBL Y		Press XEQ Y
Y0002	6378137		Value of a for WGS84/NAD83/GRS80
Y0003	STO A		
Y0004	0.006694381		Value of e^2 for WGS84/NAD83/GRS80
Y0005	STO E		
Y0006	INPUT A	6378137	Enter value of a if different; Press R/S
Y0007	INPUT E	0.006694381	Enter value of e^2 if different; Press R/S
Y0008	INPUT X		Enter X co-ordinate of point; Press R/S
Y0009	INPUT Y		Enter Y co-ordinate of point; Press R/S
Y0010	INPUT Z		Enter Z co-ordinate of point; Press R/S
Y0011	RCL Y		
Y0012	RCL X		
Y0013	÷		
Y0014	ATAN		
Y0015	STO L		
Y0016	1		
Y0017	RCL– E		
Y0018	RCL A		
Y0019	x^2		
Y0020	×		
Y0021	√x		
Y0022	STO B		
Y0023	RCL A		
Y0024	x^2		
Y0025	RCL B		
Y0026	x^2		
Y0027	–		
Y0028	RCL B		
Y0029	x^2		
Y0030	÷		
Y0031	STO D		
Y0032	RCL X		
Y0033	RCL Y		
Y0034	y, x→θ, r		
Y0035	STO P		
Y0036	RCL Z		
Y0037	x < > y		
Y0038	÷		

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Line	Instruction
Y0039	RCL A
Y0040	RCL÷ B
Y0041	×
Y0042	ATAN
Y0043	STO U
Y0044	SIN
Y0045	3
Y0046	y^x
Y0047	RCL× B
Y0048	RCL× D
Y0049	RCL+ Z
Y0050	RCL U
Y0051	COS
Y0052	3
Y0053	y^x
Y0054	RCL× A
Y0055	RCL× E
Y0056	RCL P
Y0057	x < > y
Y0058	–
Y0059	÷
Y0060	ATAN
Y0061	STO F
Y0062	RCL A
Y0063	1
Y0064	RCL F
Y0065	SIN
Y0066	x^2
Y0067	RCL× E
Y0068	–
Y0069	$\sqrt{x}$
Y0070	÷
Y0071	STO V
Y0072	RCL F
Y0073	COS
Y0074	×
Y0075	RCL L
Y0076	COS
Y0077	×
Y0078	STO C
Y0079	RCL– X
Y0080	x^2
Y0081	STO H
Y0082	RCL C
Y0083	x^2

Line	Instruction
Y0084	STO G
Y0085	RCL C
Y0086	RCL L
Y0087	TAN
Y0088	×
Y0089	STO C
Y0090	x^2
Y0091	STO+ G
Y0092	RCL C
Y0093	RCL– Y
Y0094	x^2
Y0095	STO+ H
Y0096	RCL V
Y0097	1
Y0098	RCL– E
Y0099	×
Y0100	RCL F
Y0101	SIN
Y0102	×
Y0103	STO C
Y0104	x^2
Y0105	STO+ G
Y0106	RCL C
Y0107	RCL– Z
Y0108	x^2
Y0109	STO+ H
Y0110	RCL X
Y0111	x^2
Y0112	RCL Y
Y0113	x^2
Y0114	+
Y0115	RCL Z
Y0116	x^2
Y0117	+
Y0118	$\sqrt{x}$
Y0019	RCL G
Y0120	$\sqrt{x}$
Y0121	–
Y0122	ENTER
Y0123	ABS
Y0124	÷
Y0125	RCL H
Y0126	$\sqrt{x}$
Y0127	×
Y0128	STO H

Line	Instruction
Y0129	RCL F
Y0130	→HMS
Y0131	STO F
Y0132	VIEW F
Y0133	RCL L
Y0134	→HMS
Y0135	STO L
Y0136	VIEW L
Y0137	VIEW H
Y0138	RTN

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Notes

(1) A program to convert X, Y, Z geocentric co-ordinates to latitude, longitude and ellipsoidal height on any ellipsoid.

(2) The assumption is that the distances are in meters, but by using feet for the semi-major axis of the ellipsoid, co-ordinates in feet can be produced.

(3) The program pre-enters the parameters for the WGS84/NAD83/GRS80 ellipsoid by default (in meters), to save you having to remember these. If you want a different ellipsoid, enter the appropriate a and e^2 values at the prompt (A and E).

(4) The resulting latitude, longitude and height are displayed with a prompt or label. Note that the program does not clear registers after use. You can get ν for the point by using the RCL V keystrokes.

(5) The latitude and longitude are displayed in HP notation, i.e., DDD.MMSS. The height is assumed to be in the same units as the semi-major axis, by default, meters.

(6) The sign convention with latitudes, longitudes and heights is the standard one, and as follows. Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in the US are negative. Heights below the ellipsoid are shown as negative.

(7) Pay particular attention to the sign of the co-ordinate values for the point. West of longitude 90° W, all X values will be negative; west of Greenwich (i.e., all of the US) all Y values are negative; south of the equator, all Z values will be negative. Incorrect signs will throw the position out rather dramatically.

(8) Owing to rounding in the calculator, it is possible for a value like 80° 00' 00" to be displayed as 79.5960, rather than 80.0000. You can convert the results to the appropriate representation in your head or on paper, as there is no difference in the internal calculations or the results.

Theory

The program implements the following equations:

$$\lambda = \arctan\left(\frac{Y}{X}\right) \qquad [1]$$

$$b^2 = a^2(1 - e^2) \qquad [2]$$

$$p = \sqrt{X^2 + Y^2} \qquad [3]$$

$$\tan u = \left(\frac{Z}{p}\right)\left(\frac{a}{b}\right) \qquad [4]$$

$$\nu = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \qquad [5]$$

XYZ Geocentric Co-ordinates to Lat/Long/Ht

$$\phi = \arctan\left(\frac{Z + \varepsilon b \sin^3 u}{p - e^2 a \cos^3 u}\right) \quad [6]$$

$$\varepsilon = e'^2 = \frac{(a^2 - b^2)}{b^2} \qquad \text{(the second eccentricity)} \quad [7]$$

$$X = \nu \cos\phi \cos\lambda \quad [8]$$

$$Y = \nu \cos\phi \sin\lambda \quad [9]$$

$$Z = \nu(1 - e^2) \sin\phi \quad [10]$$

Equation [1] provides a direct solution for the longitude, λ. The program then computes a variety of intermediate results, before using equation [6] to compute the latitude. This is a direct solution. The program then computes the X, Y, Z location for the point on the ellipsoid at ϕ, λ, using equations [8] to [10]. The distance between this point and the given X, Y Z co-ordinates is computed to determine *h*, the ellipsoidal height.

The distances to the Earth's center from the original X, Y, Z co-ordinates and the point on the ellipsoid are computed to get the correct sign for *h*, i.e., is the point above or below the surface of the ellipsoid.

The equations are from Bowering (1976).

Sample Computation

Inputs	$a = 6\,378\,137$ m	
	$e^2 = 0.006\,694\,381$	(WGS84/NAD83/GRS80 parameters)
	$X = 1\,353\,776.483$ m	
	$Y = -5\,052\,362.616$ m	
	$Z = 3\,637\,981.622$ m	
Results	ϕ = 35° 00' 00" N	(displayed as F = 35.000000)
	λ = 75° 00' 00" W	(displayed as L = –75.000000)
	h = 200.000 m	(displayed as H = 200.0000049)

Note that the precision of the answer depends upon the precision of the input. One millimeter (0.001m) at the surface of the ellipsoid equates to 0.00003" of arc of latitude and generally a smaller amount of longitude. Consider your input precisions and adjust the quoted precision of the outputs to match.

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Running the Program

Begin by pressing XEQ Y

The calculator displays: A?
6,378,137.0000

This is the NAD83/WGS84/GRS80 ellipsoid semi-major axis. If this is OK, press R/S; if not key in correct value and press R/S.

The calculator displays: E?
0.006694381 (suitably rounded, according to your settings)

This is the eccentricity of the NAD83/WGS84/GRS80 ellipsoid, e^2. If this OK, press R/S; if not, key in correct value and press R/S.

The calculator displays: X?
[*Whatever value happens to be in this register*]

Key in the X co-ordinate of the point and press R/S. Remember to use the appropriate sign, if negative. In the given example, key in 1353776.483 and press R/S.

The calculator displays: Y?
[*Whatever value happens to be in this register*]

Key in the Y co-ordinate of the point and press R/S. Remember to use the appropriate sign, if negative. In the given example, key in –5052362.616 and press R/S.

The calculator displays: Z?
[*Whatever value happens to be in this register*]

Key in the Z co-ordinate for the point and press R/S. Remember to use the appropriate sign, if negative. In the given example, key in 3637981.622 and press R/S.

The calculator displays RUNNING for a short while.

The calculator displays: F=
35.00000000 Press R/S

The calculator displays L=
–75.0000000 Press R/S

The calculator displays: H=
200.0000049

These calculations agree with the NGS website computations to within 0.001 m in height and 0.00002" in latitude and longitude.

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Storage Registers Used

A Semi-major axis of the ellipsoid, a. By default, set to 6378137 m.

B Semi-minor axis of the ellipsoid. Computed from a and e^2.

C Intermediate results.

D Second eccentricity of ellipsoid.

E Eccentricity of the ellipsoid, e^2. By default, set to 0.006694381.

F Latitude (geodetic) of the point, ϕ.

G Distance from ellipsoid point to center of Earth.

H Ellipsoidal height of the point, *h*.

L Longitude of the point, λ.

P Intermediate value.

U Intermediate value.

V The radius of curvature of the ellipsoid in the prime vertical, ν.

X Geocentric X co-ordinate of the point.

Y Geocentric Y co-ordinate of the point.

Z Geocentric Z co-ordinate of the point.

Note that this program overwrites the Z register, removing the value of 360 that some other programs use for correcting negative angles. Check this if using one of the closure programs.

Labels Used

Label **Y** Length = 492 Checksum = 3347

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

References

Bowering, B.R., 1976. Transformation from spatial to geographical co-ordinates. *Survey Review*, No. 181, pp. 323–327.

The NGS website for the interactive XYZ ⇔ lat/long/height converter:

http://www.ngs.noaa.gov/TOOLS/XYZ/xyz.shtml

History

The original version (v. 1) implemented an iterated solution for the latitude and height. This version (v. 2) uses a direct solution. It requires 45 more lines of code, but produces a result faster and without the vagaries of iterations.