HP-35s Calculator Program XYZ 2

Convert XYZ Geocentric Co-ordinates on any ellipsoid to Latitude, Longitude and Height

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Date: October, 2007. **Mnemonic**: **L** for 'XYZ to **L**at/**L**ong'

Line	Instruction	Display	User Instructions
L001	LBL L		Press XEQ L ENTER to run program
L002	CLSTK		
L003	FS? 10		
L004	GTO L008		
L005	SF 1		
L006	SF 10		
L007	GTO L009		
L008	CF 1		
L009	XYZ TO LATLONG		
L010	PSE		
L011	6378137		Value of a for WGS84/NAD83/GRS80
L012	STO A		
L013	0.006694381		Value of e^2 for WGS84/NAD83/GRS80
L014	STO E		
L015	SEMIMAJOR AXIS		
L016	PSE		
L017	INPUT A	6378137	Enter value of a if different; Press R/S
L018	E SQUARED		
L019	PSE		
L020	INPUT E	0.006694381	Enter value of e^2 if different; Press R/S
L021	ENTER X CO-ORD		
L022	PSE		
L023	INPUT X		Enter X co-ordinate of point; Press R/S
L024	ENTER Y CO-ORD		
L025	PSE		
L026	INPUT Y		Enter Y co-ordinate of point; Press R/S
L027	ENTER Z CO-ORD		
L028	PSE		
L029	INPUT Z		Enter Z co-ordinate of point; Press R/S
L030	RCL Y		
L031	RCL X		
L032	÷		
L033	ATAN		
L034	STO L		
L035	1		
L036	RCL– E		
L037	RCL A		
L038	x^2		

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Line	Instruction
L039	×
L040	√x
L041	STO B
L042	RCL A
L043	x^2
L044	RCL B
L045	x^2
L046	–
L047	RCL B
L048	x^2
L049	÷
L050	STO D
L051	RCL X
L052	x^2
L053	RCL Y
L054	x^2
L055	+
L056	√x
L057	STO P
L058	RCL Z
L059	x < > y
L060	÷
L061	RCL A
L062	RCL÷ B
L063	×
L064	ATAN
L065	STO U
L066	SIN
L067	3
L068	y^x
L069	RCL× B
L070	RCL× D
L071	RCL+ Z
L072	RCL U
L073	COS
L074	3
L075	y^x
L076	RCL× A
L077	RCL× E
L078	RCL P
L079	x < > y
L080	–
L081	÷
L082	ATAN
L083	STO F

Line	Instruction
L084	RCL A
L085	1
L086	RCL F
L087	SIN
L088	x^2
L089	RCL× E
L090	–
L091	√x
L092	÷
L093	STO V
L094	RCL F
L095	COS
L096	×
L097	RCL L
L098	COS
L099	×
L100	STO C
L101	RCL– X
L102	x^2
L103	STO H
L104	RCL C
L105	x^2
L106	STO G
L107	RCL C
L108	RCL L
L109	TAN
L110	×
L111	STO C
L112	x^2
L113	STO+ G
L114	RCL C
L115	RCL– Y
L116	x^2
L117	STO+ H
L118	RCL V
L019	1
L120	RCL– E
L121	×
L122	RCL F
L123	SIN
L124	×
L125	STO C
L126	x^2
L127	STO+ G
L128	RCL C

Line	Instruction
L129	RCL– Z
L130	x^2
L131	STO+ H
L132	RCL X
L133	x^2
L134	RCL Y
L135	x^2
L136	+
L137	RCL Z
L138	x^2
L139	+
L140	√x
L141	RCL G
L142	√x
L143	–
L144	ENTER
L145	ABS
L146	÷
L147	RCL H
L148	√x
L149	×
L150	STO H
L151	RCL F
L152	→HMS
L153	STO F
L154	LATITUDE
L155	PSE
L156	VIEW F
L157	RCL L
L158	→HMS
L159	STO L
L160	LONGITUDE
L161	PSE
L162	VIEW L
L163	ELLIPSE HEIGHT
L164	PSE
L165	VIEW H
L166	RCL H
L167	RCL L
L168	RCL F
L169	FS? 1
L170	CF 10
L171	RTN

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Notes

(1) A program to convert X, Y, Z geocentric co-ordinates to latitude, longitude and ellipsoidal height on any ellipsoid.

(2) The assumption is that the distances are in meters, but by using feet for the semi-major axis of the ellipsoid, co-ordinates in feet can be produced.

(3) The program pre-enters the parameters for the WGS84/NAD83/GRS80 ellipsoid by default (in meters), to save you having to remember these. If you want a different ellipsoid, enter the appropriate a and e^2 values at the prompt (A and E). To use the provided values, just press R/S when they appear.

(4) The resulting latitude, longitude and height are displayed with a prompt or label. Note that the program does not clear registers after use. You can get ν for the point by using the RCL V keystrokes.

(5) The latitude and longitude are displayed in HP notation, i.e., DDD.MMSS. The height is assumed to be in the same units as the semi-major axis, by default, meters.

(6) The sign convention with latitudes, longitudes and heights is the standard one, as follows. Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in the US are negative. Heights below the ellipsoid are shown as negative.

(7) Pay particular attention to the sign of the co-ordinate values for the point. West of longitude 90° W, all X values will be negative; west of Greenwich (i.e., all of the US) all Y values are negative; south of the equator, all Z values will be negative. Incorrect signs will throw the position out rather dramatically.

(8) Owing to rounding in the calculator, it is possible for a value like 80° 00' 00" to be displayed as 79.5960, rather than 80.0000. You can convert the results to the appropriate representation in your head or on paper, as there is no difference in the internal calculations or the results.

(9) When the program finishes, it places the computed values on the stack. The height of the point above the ellipsoid is in the Z stack register. The longitude (in HP notation) is in the Y stack register. The latitude (in HP notation) is in the X stack register. This allows the program to be called from another program (as a sub-routine) and return values on the stack for further processing.

(10) The program sets Flag 10, to allow prompts to be displayed. When the program finishes, it sets Flag 10 back to its original value. To do this, the program uses Flag 1.

Theory

The program implements the following equations:

$$\lambda = \arctan\left(\frac{Y}{X}\right) \qquad [1]$$

$$b^2 = a^2(1 - e^2) \qquad \text{[the semi-minor axis length]} \qquad [2]$$

XYZ Geocentric Co-ordinates to Lat/Long/Ht

$$p = \sqrt{X^2 + Y^2} \quad [3]$$

$$\tan u = \left(\frac{Z}{p}\right)\left(\frac{a}{b}\right) \quad [4]$$

$$\nu = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \quad [5]$$

$$\phi = \arctan\left(\frac{Z + \varepsilon b \sin^3 u}{p - e^2 a \cos^3 u}\right) \quad [6]$$

$$\varepsilon = e'^2 = \frac{(a^2 - b^2)}{b^2} \quad \text{[the second eccentricity]} \quad [7]$$

$$X = \nu \cos\phi \cos\lambda \quad [8]$$

$$Y = \nu \cos\phi \sin\lambda \quad [9]$$

$$Z = \nu(1 - e^2) \sin\phi \quad [10]$$

Equation [1] provides a direct solution for the longitude, λ. The program then computes a variety of intermediate results, before using equation [6] to compute the latitude. This is a direct solution. The program then computes the X, Y, Z location for the point on the ellipsoid at ϕ, λ, using equations [8] to [10]. The distance between this point and the given X, Y Z co-ordinates is computed to determine *h*, the ellipsoidal height.

The distances to the Earth's center from the original X, Y, Z co-ordinates and the point on the ellipsoid are computed to get the correct sign for *h*, i.e., is the point above or below the surface of the ellipsoid.

The equations are from Bowering (1976).

Sample Computation

Inputs

a = 6 378 137 m
e^2 = 0.006 694 381 (WGS84/NAD83/GRS80 parameters)
X = 1 353 776.483 m
Y = –5 052 362.616 m
Z = 3 637 981.622 m

Results

ϕ = 35° 00' 00" N (displayed as F = 35.000000)
λ = 75° 00' 00" W (displayed as L = –75.000000)
h = 200.000 m (displayed as H = 200.0000049)

Note that the precision of the answer depends upon the precision of the input. One millimeter (0.001m) at the surface of the ellipsoid equates to 0.00003" of arc of latitude and generally a smaller amount of longitude. Consider your input precisions and adjust the quoted precision of the outputs to match. You can use the SHOW function to see all the digits in a number.

XYZ Geocentric Co-ordinates to Lat/Long/Ht

Running the Program

Begin by pressing XEQ L ENTER

The calculator displays XYZ to LATLONG, briefly, then displays SEMIMAJOR AXIS, briefly.

The calculator displays: A?
6,378,137.0000

This is the NAD83/WGS84/GRS80 ellipsoid semi-major axis. If this is OK, press R/S; if not key in correct value and press R/S.

The calculator displays: E SQUARED, briefly.
E?
0.006694381 (suitably rounded, according to your settings)

This is the eccentricity of the NAD83/WGS84/GRS80 ellipsoid, e^2. If this OK, press R/S; if not, key in correct value and press R/S.

The calculator displays: ENTER X COORD, briefly.
X?
[*Whatever value happens to be in this register*]

Key in the X co-ordinate of the point and press R/S. Remember to use the appropriate sign, if negative. In the given example, key in 1353776.483 and press R/S.

The calculator displays: ENTER Y COORD, briefly.
Y?
[*Whatever value happens to be in this register*]

Key in the Y co-ordinate of the point and press R/S. Remember to use the appropriate sign, if negative. In the given example, key in –5052362.616 and press R/S.

The calculator displays: ENTER Z COORD, briefly.
Z?
[*Whatever value happens to be in this register*]

Key in the Z co-ordinate for the point and press R/S. Remember to use the appropriate sign, if negative. In the given example, key in 3637981.622 and press R/S.

The calculator displays RUNNING for a short while.

The calculator displays: LATITUDE, briefly.
F=
35.00000000 Press R/S

The calculator display LONGITUDE, briefly.
L=
–75.0000000 Press R/S

The calculator displays: ELLIPSE HEIGHT, briefly.
H=
200.0000049 Press R/S

The calculator now completes its run, placing the latitude, longitude and height on the stack, and resetting Flag 10 to its state when the program started.

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These calculations agree with the NGS website computations to within 0.001 m in height and 0.00002" in latitude and longitude.

Storage Registers Used

A Semi-major axis of the ellipsoid, a. By default, set to 6378137 m.

B Semi-minor axis of the ellipsoid. Computed from a and e^2.

C Intermediate value.

D Second eccentricity of ellipsoid.

E Eccentricity of the ellipsoid, e^2. By default, set to 0.006694381.

F Latitude (geodetic) of the point, ϕ.

G Distance from ellipsoid point to center of Earth.

H Ellipsoidal height of the point, *h*.

L Longitude of the point, λ.

P Intermediate value.

U Intermediate value.

V The radius of curvature of the ellipsoid in the prime vertical, ν.

X Geocentric X co-ordinate of the point.

Y Geocentric Y co-ordinate of the point.

Z Geocentric Z co-ordinate of the point.

Labels Used

Label **L** Length = 643 Checksum = 36FE

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

References

Bowering, B.R., 1976. Transformation from spatial to geographical co-ordinates. *Survey Review*, No. 181, pp. 323–327.

The NGS website for the interactive XYZ $\Leftrightarrow$ lat/long/height converter:

http://www.ngs.noaa.gov/TOOLS/XYZ/xyz.shtml