HP-35s Calculator Program
Convert Latitude, Longitude and Height on any ellipsoid to XYZ Geocentric Co-ordinates

Programmer: Dr. Bill Hazelton
Date: October, 2007. Mnemonic: G for 'lat/long to Geocentric co-ordinates.'

| Line | Instruction | Display | User Instructions |
| :---: | :---: | :---: | :---: |
| G001 | LBL G |  | Press XEQ G ENTER to run program |
| G002 | CLSTK |  |  |
| G003 | FS? 10 |  |  |
| G004 | GTO G008 |  |  |
| G005 | SF 1 |  |  |
| G006 | SF 10 |  |  |
| G007 | GTO G009 |  |  |
| G008 | CF 1 |  |  |
| G009 | LATLONG TO XYZ |  |  |
| G010 | PSE |  |  |
| G011 | 6378137 |  | Value of a for WGS84/NAD83/GRS80 |
| G012 | STO A |  |  |
| G013 | 0.006694381 |  | Value of $\mathrm{e}^{2}$ for WGS84/NAD83/GRS80 |
| G014 | STO E |  |  |
| G015 | SEMIMAJOR AXIS |  |  |
| G016 | PSE |  |  |
| G017 | INPUT A | 6378137 | Enter value of a if different |
| G018 | E SQUARED |  |  |
| G019 | PSE |  |  |
| G020 | INPUT E | 0.006694381 | Enter value of $\mathrm{e}^{2}$ if different |
| G021 | ENTER LATITUDE |  |  |
| G022 | PSE |  |  |
| G023 | INPUT F |  | Enter $\phi$ of point (DDD.MMSSss) |
| G024 | ENTR LONGITUDE |  |  |
| G025 | PSE |  |  |
| G026 | INPUT L |  | Enter $\boldsymbol{\lambda}$ of point (DDD.MMSSss) |
| G027 | ENTER HEIGHT |  |  |
| G028 | PSE |  |  |
| G029 | INPUT H |  | Enter h of point |
| G030 | RCL A |  |  |
| G031 | 1 |  |  |
| G032 | RCL F |  |  |
| G033 | HMS $\rightarrow$ |  |  |
| G034 | SIN |  |  |
| G035 | $\mathrm{x}^{2}$ |  |  |
| G036 | RCL× E |  |  |
| G037 | - |  |  |
| G038 | $\sqrt{x}$ |  |  |

Lat/Long/Ht to XYZ Geocentric Co-ordinates

| Line | Instruction | Display |
| :--- | :--- | :--- |

## Notes

(1) A program to convert latitude, longitude and ellipsoidal height on any ellipsoid to $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ geocentric co-ordinates.
(2) The assumption is that the distances are in meters, but by using feet for the semimajor axis of the ellipsoid, co-ordinates in feet will be produced.
(3) The program pre-enters the parameters for the WGS84/NAD83/GRS80 ellipsoid by default (in meters), to save you having to remember these. If you want a different ellipsoid, enter the appropriate $a$ and $\mathrm{e}^{2}$ values at the prompts ( A and E ).
(4) The resulting co-ordinates are displayed with a prompt or label. Note that the program does not clear registers after use. You can get $v$ for the point by using the RCL V keystrokes, for example.
(5) The latitude and longitude are entered in HP notation, i.e., DDD.MMSS. The height is assumed to be in the same units as the semi-major axis, by default, meters.
(6) It is critical to follow the sign convention with latitudes, longitudes and heights. Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in the US are negative. Heights below the ellipsoid must be entered as negative.
(7) The program sets Flag 10 to allow display of equations as prompts. However, it stores the state of Flag 10 (in Flag 1), and restores the original state after the program ends. If you don't finish the program, you may need to check and reset Flag 10.
(8) When the program ends, it leaves the $\mathrm{X}, \mathrm{Y}$ and Z values on the stack, in the $\mathrm{X}, \mathrm{Y}$ and Z stack registers, respectively. They can then be used by another program. The input values are still in the original storage registers (in HP angle notation for latitude and longitude), and can be recalled later or by other programs. This program may be called by another program as a sub-routine, and will return the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ values on the stack.

## Theory

The program implements the following four equations:

$$
\begin{align*}
& \mathrm{X}=(v+h) \cos \phi \cos \lambda  \tag{1}\\
& \mathrm{Y}=(v+h) \cos \phi \sin \lambda  \tag{2}\\
& \mathrm{Z}=\left[v\left(1-\mathrm{e}^{2}\right)+h\right] \sin \phi  \tag{3}\\
& \mathrm{v}=\frac{a}{\sqrt{1-e^{2} \sin ^{2} \phi}} \tag{4}
\end{align*}
$$

These provide a direct solution. Values for the ellipsoid (a and $\mathrm{e}^{2}$ ) are requested, although default values for NAD83/WGS84/GRS80 are supplied (these can be overwritten). The latitude, longitude and ellipsoidal height of the point are requested of the user. The program supplies the solution in three pieces.

## Sample Computation

Inputs
$\mathrm{a}=6378137 \mathrm{~m}$
$\mathrm{e}^{2}=0.006694381 \quad$ (WGS84/NAD83/GRS80 parameters)
$\phi=35^{\circ} 00^{\prime} 00^{\prime \prime} \mathrm{N}$ (entered as 35.0000 ) Latitude of point
$\lambda=75^{\circ} 00^{\prime} 00^{\prime \prime} \mathrm{W}$ (entered as -75.0000 ) Longitude of point
$\mathrm{h}=200 \mathrm{~m} \quad$ Ellipsoidal height of point

Results

$$
\begin{aligned}
\mathrm{X} & = \\
\mathrm{Y} & =-5353776.483 \mathrm{~m} \\
\mathrm{Z} & =363362.616 \mathrm{~m} \\
& 3637981.622 \mathrm{~m}
\end{aligned}
$$

## Running the Program

Begin by pressing XEQ G ENTER to start the program. The calculator briefly displays LATLONG TO XYZ, then SEMIMAJOR AXIS, briefly.

The calculator displays:

A?
6,378,137.0000

This is the NAD83/WGS84/GRS80 ellipsoid semi-major axis. If this is OK, press R/S; if not key in correct value and press R/S. The calculator displays E SQUARED, briefly.

The calculator displays: E? 0.006694381 (suitably rounded, according to your settings)

This is the eccentricity of the NAD83/WGS84/GRS80 ellipsoid, $\mathrm{e}^{2}$. If this OK, press R/S; if not, key in correct value and press R/S. The calculator displays ENTER LATITUDE, briefly.

The calculator displays:

F?
[Whatever value happens to be in this register]

Key in the latitude of the point and press R/S. Use negative values for the southern hemisphere. In the given example, key in 35 and press R/S. The calculator displays ENTR LONGITUDE, briefly.

The calculator displays: L?
[Whatever value happens to be in this register]
Key in the longitude of the point and press R/S. Use negative values in the western hemisphere. In the given example, key in -75 and press R/S. The calculator displays ENTER HEIGHT, briefly.

The calculator displays: $\quad \mathrm{H}$ ?
[Whatever value happens to be in this register]
Key in the ellipsoidal height for the point and press R/S. Use negative values for heights below the ellipsoid. In the given example, key in 200 and press R/S.

The calculator displays:
GEOCENTRIC X, briefly, then:
$\mathrm{X}=$

$$
1,353,776.4829 \quad \text { Press R/S }
$$

The calculator displays: GEOCENTRIC Y, briefly, then:
$\mathrm{Y}=$
$-5,052,362.6164$
Press R/S

## Lat/Long/Ht to XYZ Geocentric Co-ordinates

The calculator displays: GEOCENTRIC Z, briefly, then:
Z=
3,637,981.6216 Press R/S

The calculator now completes the program, resetting Flag 10, and placing the computed values in the stack. The stack now holds the following values:

## Stack Level Contents

T Geocentric Z Co-ordinate
Z Geocentric Z Co-ordinate
Y Geocentric Y Co-ordinate
X Geocentric X Co-ordinate
These calculations agree with the NGS website computations to within 0.003 m .

## Storage Registers Used

A Semi-major axis of the ellipsoid, $a$. By default, set to 6378137 m
E Squared eccentricity of the ellipsoid, $e^{2}$. By default, set to 0.006694381 .
F Latitude (geodetic) of the point, $\phi$ (in HP notation).
H Ellipsoidal height of the point, h .
L Longitude of the point, $\lambda$ (in HP notation).
V The radius of curvature of the ellipsoid in the prime vertical, $v$.
$\mathbf{X} \quad$ Geocentric X co-ordinate of the point.
$\mathbf{Y} \quad$ Geocentric Y co-ordinate of the point.
$\mathbf{Z} \quad$ Geocentric Z co-ordinate of the point.

## Labels Used

Label G

$$
\text { Length }=373 \quad \text { Checksum }=77 \mathrm{D} 4
$$

Use the length ( $\mathrm{LN}=$ ) and Checksum ( $\mathrm{CK}=$ ) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Note that the checksum depends upon entering prompts exactly as written in the program listing.

## Reference

The NGS website for the interactive $\mathrm{XYZ} \Leftrightarrow$ lat/long/height converter:
http://www.ngs.noaa.gov/TOOLS/XYZ/xyz.shtml

