## Convert Latitude, Longitude and Height on any ellipsoid to XYZ Geocentric Co-ordinates

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Date: October, 2007. Mnemonic: G for 'lat/long to Geocentric co-ordinates.'

Line	Instruction	Display	User Instructions
G001	LBL G		Press XEQ G ENTER to run program
G002	CLSTK		
G003	FS? 10		
G004	GTO G008		
G005	SF 1		
G006	SF 10		
G007	GTO G009		
G008	CF 1		
G009	LATLONG TO XYZ		
G010	PSE		
G011	6378137		Value of a for WGS84/NAD83/GRS80
G012	STO A		
G013	0.006694381		Value of e <sup>2</sup> for WGS84/NAD83/GRS80
G014	STO E		
G015	SEMIMAJOR AXIS		
G016	PSE		
G017	INPUT A	6378137	Enter value of a if different
G018	E SQUARED		
G019	PSE		
G020	INPUT E	0.006694381	Enter value of $e^2$ if different
G021	ENTER LATITUDE		
G022	PSE		
G023	INPUT F		Enter $\phi$ of point (DDD.MMSSss)
G024	ENTR LONGITUDE		
G025	PSE		
G026	INPUT L		Enter $\lambda$ of point (DDD.MMSSss)
G027	ENTER HEIGHT		
G028	PSE		
G029	INPUT H		Enter h of point
G030	RCL A		
G031	1		
G032	RCL F		
G033	HMS→		
G034	SIN		
G035	$\mathbf{x}^2$		
G036	RCL× E		
G037	-		
G038	$\sqrt{\mathbf{x}}$		

#### HP-35s Calculator Program

### Lat/Long/Ht to XYZ Geocentric Co-ordinates

Line	Instruction	Display	User Instructions
G039	÷		
G040	STO V		
G041	RCL+ H		
G042	RCL F		
G043	HMS→		
G044	COS		
G045	X		
G046	RCL L		
G047	HMS→		
G048	COS		
G049	X		
G050	STO X		
G051	GEOCENTRIC X		
G052	PSE		
G053	VIEW X	X co-ordinate	X co-ordinate is displayed
G054	RCL L		
G055	HMS→		
G056	TAN		
G057	×		
G058	STO Y		
G059	GEOCENTRIC Y		
G060	PSE		
G061	VIEW Y	Y co-ordinate	Y co-ordinate is displayed
G062	RCL V		
G063	1		
G064	RCL- E		
G065	X		
G066	RCL+ H		
G067	RCL F		
G068	$HMS \rightarrow$		
G069	SIN		
G070	×		
G071	STO Z		
G072	GEOCENTRIC Z		
G073	PSE		
G074	VIEW Z	Z co-ordinate	Z co-ordinate is displayed
G075	FS? 1		
G076	CF 10		
G077	RCL Z		
G078	RCL Y		
G079	RCL X		X, Y, Z values left on stack
G080	RTN		Program ends

#### Lat/Long/Ht to XYZ Geocentric Co-ordinates

#### Notes

- (1) A program to convert latitude, longitude and ellipsoidal height on any ellipsoid to X, Y, Z geocentric co-ordinates.
- (2) The assumption is that the distances are in meters, but by using feet for the semimajor axis of the ellipsoid, co-ordinates in feet will be produced.
- (3) The program pre-enters the parameters for the WGS84/NAD83/GRS80 ellipsoid by default (in meters), to save you having to remember these. If you want a different ellipsoid, enter the appropriate a and  $e^2$  values at the prompts (A and E).
- (4) The resulting co-ordinates are displayed with a prompt or label. Note that the program does not clear registers after use. You can get v for the point by using the RCL V keystrokes, for example.
- (5) The latitude and longitude are entered in HP notation, i.e., DDD.MMSS. The height is assumed to be in the same units as the semi-major axis, by default, meters.
- (6) It is critical to follow the sign convention with latitudes, longitudes and heights. Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in the US are negative. Heights below the ellipsoid must be entered as negative.
- (7) The program sets Flag 10 to allow display of equations as prompts. However, it stores the state of Flag 10 (in Flag 1), and restores the original state after the program ends. If you don't finish the program, you may need to check and reset Flag 10.
- (8) When the program ends, it leaves the X, Y and Z values on the stack, in the X, Y and Z stack registers, respectively. They can then be used by another program. The input values are still in the original storage registers (in HP angle notation for latitude and longitude), and can be recalled later or by other programs. This program may be called by another program as a sub-routine, and will return the X, Y, Z values on the stack.

#### Theory

The program implements the following four equations:

$$X = (v + h) \cos \phi \, \cos \lambda \tag{1}$$

$$Y = (v + h) \cos \phi \sin \lambda$$
 [2]

$$Z = [v(1 - e^2) + h] \sin \phi$$
[3]

$$\mathbf{v} = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \tag{4}$$

These provide a direct solution. Values for the ellipsoid (a and  $e^2$ ) are requested, although default values for NAD83/WGS84/GRS80 are supplied (these can be overwritten). The latitude, longitude and ellipsoidal height of the point are requested of the user. The program supplies the solution in three pieces.

Lat/Long/Ht to XYZ Geocentric Co-ordinates

#### Sample Computation

Inputs	$      a = 6 378 137 m \\ e^2 = 0.006 694 381 \qquad (WGS84/NAD83/GRS80 parameters) \\ \phi = 35^\circ 00' 00" N \text{ (entered as 35.0000) Latitude of point} \\ \lambda = 75^\circ 00' 00" W \text{ (entered as -75.0000) Longitude of point} \\ h = 200 m \qquad \qquad$
Results	$\begin{array}{rcl} X & = & 1 \ 353 \ 776.483 \ m \\ Y & = & -5 \ 052 \ 362.616 \ m \\ Z & = & 3 \ 637 \ 981.622 \ m \end{array}$
Running the Program	l
Begin by pressing XEQ LATLONG TO XYZ, then S	G ENTER to start the program. The calculator briefly displays EMIMAJOR AXIS, briefly.
The calculator displays:	A?

This is the NAD83/WGS84/GRS80 ellipsoid semi-major axis. If this is OK, press R/S; if not key in correct value and press R/S. The calculator displays E SQUARED, briefly.

The calculator displays:	E? 0.006694381	(suitably rounded, according to your settings)
		• • • •

6.378.137.0000

**F**?

This is the eccentricity of the NAD83/WGS84/GRS80 ellipsoid, e<sup>2</sup>. If this OK, press R/S; if not, key in correct value and press R/S. The calculator displays ENTER LATITUDE, briefly.

The calculator displays:

[Whatever value happens to be in this register]

Key in the latitude of the point and press R/S. Use negative values for the southern hemisphere. In the given example, key in 35 and press R/S. The calculator displays ENTR LONGITUDE, briefly.

The calculator displays: L?

[Whatever value happens to be in this register]

Key in the longitude of the point and press R/S. Use negative values in the western hemisphere. In the given example, key in -75 and press R/S. The calculator displays ENTER HEIGHT, briefly.

The calculator displays:H?<br/>[Whatever value happens to be in this register]

Key in the ellipsoidal height for the point and press R/S. Use negative values for heights below the ellipsoid. In the given example, key in 200 and press R/S.

The calculator displays:	GEOCENTRIC X, briefly, then: X=		
	1,353,776.4829	Press R/S	
The calculator displays:	GEOCENTRIC Y, briefly, the Y=	en:	
	-5.052.362.6164	Press R/S	

# HP-35s Calculator Program Lat/Long/Ht to XYZ Geocentric Co-ordinates

The calculator displays:

GEOCENTRIC Z, briefly, then: Z= 3,637,981.6216 Press R/S XYZ 1

The calculator now completes the program, resetting Flag 10, and placing the computed values in the stack. The stack now holds the following values:

Stack Level	Contents
Т	Geocentric Z Co-ordinate
Z	Geocentric Z Co-ordinate
Y	Geocentric Y Co-ordinate
Х	Geocentric X Co-ordinate

These calculations agree with the NGS website computations to within 0.003 m.

#### **Storage Registers Used**

- A Semi-major axis of the ellipsoid, *a*. By default, set to 6378137 m
- **E** Squared eccentricity of the ellipsoid,  $e^2$ . By default, set to 0.006694381.
- **F** Latitude (geodetic) of the point,  $\phi$  (in HP notation).
- **H** Ellipsoidal height of the point, h.
- L Longitude of the point,  $\lambda$  (in HP notation).
- V The radius of curvature of the ellipsoid in the prime vertical, v.
- **X** Geocentric X co-ordinate of the point.
- Y Geocentric Y co-ordinate of the point.
- **Z** Geocentric Z co-ordinate of the point.

#### Labels Used

Label G Length = 373 Checksum = 77D4

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Note that the checksum depends upon entering prompts exactly as written in the program listing.

#### Reference

The NGS website for the interactive  $XYZ \Leftrightarrow lat/long/height converter:$ 

http://www.ngs.noaa.gov/TOOLS/XYZ/xyz.shtml