HP-35s Calculator Program OM 1

Convert Oblique Mercator Co-ordinates (SPCS, etc.) to Latitude and Longitude and vice versa

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Date: April, 2008. **Version**: 1.0 **Mnemonic**: **O** for **O**blique Mercator

Line	Instruction	Display	User Instructions
O001	LBL O		↱ LBL O
O002	CLSTK		↱ CLEAR 5
O003	FS? 10		↰ FLAGS 3 .0
O004	GTO O008		
O005	SF 1		↰ FLAGS 1 1
O006	SF 10		↰ FLAGS 1 .0
O007	GTO O009		
O008	CF 1		↰ FLAGS 2 1
O009	OBLIQUE MERCTR		(Key in using EQN RCL T, RCL M, etc.)
O010	PSE		↱ PSE
O011	CL x		↱ CLEAR 1
O012	6378137		a value for ellipsoid (WGS84/GRS80/NAD83)
O013	STO A		↱ STO A
O014	0.0066943800229034		e^2 value for ellipsoid (WGS84/GRS80/NAD83)
O015	STO E		↱ STO E
O016	0.9999		k_c value for zone (AK 1)
O017	STO K		↱ STO K
O018	–133.4		λ_c for zone (AK 1)
O019	STO L		↱ STO L
O020	57		ϕ_c for zone (AK 1)
O021	STO F		↱ STO F
O022	5000000		E_0 for zone (AK 1)
O023	STO R		↱ STO R
O024	–5000000		N_0 for zone (AK 1)
O025	STO S		↱ STO S
O026	–0.75		α_c for zone (AK 1)
O027	STO M		↱ STO M
O028	CHECK–ENTER A		(Key in using EQN RCL C, RCL H, etc.)
O029	PSE		↱ PSE
O030	INPUT A		↰ INPUT A
O031	CHECK–ENTER E		(Key in using EQN RCL C, RCL H, etc.)
O032	PSE		↱ PSE
O033	INPUT E		↰ INPUT E
O034	CHECK–ENTER K		(Key in using EQN RCL C, RCL H, etc.)
O035	PSE		↱ PSE
O036	INPUT K		↰ INPUT K
O037	CHK–NTR LONG C		(Key in using EQN RCL C, RCL H, etc.)
O038	PSE		↱ PSE

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Line	Instruction
O039	INPUT L
O040	CHK–NTR LAT 0
O041	PSE
O042	INPUT F
O043	CHK–NTR E 0
O044	PSE
O045	INPUT R
O046	CHK–NTR N 0
O047	PSE
O048	INPUT S
O049	CHK-NTR TAN AC
O050	PSE
O051	INPUT M
O052	RCL L
O053	HMS→
O054	STO L
O055	RCL F
O056	HMS→
O057	STO F
O058	RCL M
O059	ATAN
O060	STO M
O061	X—Y IN [0—1]
O062	PSE
O063	0
O064	STO Z
O065	INPUT Z
****	Set up zone values
O066	CF 10
O067	SQRT(1 + E ÷ (1 – E) × COS(F)^4) ➤ B
O068	SQRT(1 – E × SIN(F)^) ➤ W
O069	A × B × SQRT(1 – E) ÷ W^2 ➤ P
O070	(LN((1+SIN(F))÷(1–SIN(F)))–SQRT(F)×LN((1+SQRT(E)×SIN(F))÷(1–SQRT(E)×SIN(F))))÷2 ➤ Q
O071	ACOSH((B × SQRT(1 – E)) ÷ W ÷ COS(F)) – (B × Q) ➤ C
O072	K × P ÷ B ➤ D
O073	ASIN(A × SIN(M) × COS(F) ÷ P ÷ W) ➤ O
O074	K × P ÷ A ➤ I
O075	L – ASIN(SIN(O) × SINH(B × Q + C) ÷ COS(O)) ÷ B ➤ Y
O076	6 ➤ J
O077	F ➤ (J)
O078	7 ➤ J
O079	L ➤ (J)
O080	8 ➤ J

Line	Instruction
O081	M ➤ (J)
O082	9 ➤ J
O083	K ➤ (J)
O084	10 ➤ J
O085	R ➤ (J)
O086	11 ➤ J
O087	S ➤ (J)
O088	RCL Z
O089	x ≠ 0?
O090	GTO O158 //go to Convert E/N to lat/long//
****	Lat/Long to E/N Conversion
O091	SF 10
O092	ENTER PT LAT
O093	PSE
O094	INPUT F
O095	ENTER PT LONG
O096	PSE
O097	INPUT L
O098	CF 10
O099	HMS→ (L) ➤ L
O100	HMS→ (F) ➤ F
O101	→DEG(→RAD(L — Y) × B) ➤ H

Line	Instruction
O102	(LN((1+SIN(F))÷(1–SIN(F)))–SQRT(E)×LN((1+SQRT(E)×SIN(F))÷(1–SQRT(E)×SIN(F))))
O103	STO Q
O104	SINH(B × Q + C) ➤ J
O105	COSH(B × Q + C) ➤ K
O106	→RAD (ATAN((J × COS(O) – SIN(O) × SIN(–H)) ÷ COS(–H))) × D ➤ U
O107	LN((K – SIN(O)×J – COS(O)×SIN(–H)) ÷ (K + SIN(O)×J + COS(O)×SIN(–H))) × D÷2 ➤ V
O108	U × COS(M) – V × SIN(M) + S ➤ N
O109	U × SIN(M) + V × COS(M) + R ➤ T
O110	→HMS(ATAN((SIN(O) – J × COS(O) × SIN(–H)) ÷ (K × COS(O) × COS(–H)))—M) ➤ G
O111	I × SQRT(1 – E × SIN(F)^2) × COS(→DEG(U ÷ D)) ÷ COS(F) ÷ COS(–H) ➤ K
****	Display Results
O112	SF 10
O113	RESULTS
O114	PSE
O115	EASTING
O116	PSE
O117	VIEW T
O118	NORTHING
O119	PSE
O120	VIEW N
O121	GRID CONV
O122	PSE
O123	VIEW G
O124	PT SCALE FACT
O125	PSE
O126	VIEW K
O127	0
O128	STO Z
O129	NEXT PT [0—1]
O130	PSE
O131	INPUT Z
O132	RCL Z
O133	x = 0?
O134	GTO O294 //go to end//
O135	NEW ZONE [0—1]
O136	PSE
O137	CF 10
O138	6 ➤ J
O139	(J) ➤ F
O140	7 ➤ J
O141	(J) ➤ L
O142	8 ➤ J
O143	TAN((J)) ➤ M ‡
O144	9 ➤ J
O145	(J) ➤ K
O146	0 ➤ Z
O147	SF 10
O148	CLx
O149	INPUT Z
O150	RCL Z
O151	x = 0?
O152	GTO O058 //go to same zone//
O153	CF 10
O154	→HMS (F) ➤ F
O155	→HMS (L) ➤ L
O156	SF 10
O157	GTO O028 //go to new zone//
****	Get E/N values to convert to lat/long
O158	SF 10
O159	ENTER EASTING
O160	PSE
O161	INPUT T
O162	ENTER NORTHING
O163	PSE
O164	INPUT N
O165	CF 10

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Line	Instruction
O166	1 ➤ J //numeral 1 (one)//
O167	I ➤ (J) //letter I//
O168	(T – R) × SIN(M) + (N – S) × COS(M) ➤ U
O169	(T – R) × COS(M) – (N – S) × SIN(M) ➤ V
O170	SINH(V ÷ D) ➤ J
O171	COSH(V ÷ D) ➤ P
O172	SIN(→DEG(U ÷ D)) ➤ W
O173	(LN((P – J × SIN(O) + COS(O) × W) ÷ (P + J × SIN(O) – COS(O) × W)) ÷ 2 – C) ÷ B ➤ Q
O174	2 × ATAN((EXP(Q) – 1) ÷ (EXP(Q) + 1)) ➤ G
O175	2 ➤ I
O176	E ÷ 2 + 5 ÷ 24 × E^2 + E^3 ÷ 12 + 13 ÷ 360 × E^4 ➤ (I)
O177	3 ➤ I
O178	7 ÷ 48 × E^2 + 29 ÷ 240 × E^3 + 811 ÷ 11520 × E^4 ➤ (I)
O179	4 ➤ I
O180	7 ÷ 120 × E^3 + 81 ÷ 1120 × E^4 ➤ (I)
O181	5 ➤ I
O182	4279 ÷ 161280 × E^4 ➤ (I)
O183	2 ➤ I
O184	(I) ➤ T
O185	3 ➤ I
O186	(I) × 2
O187	STO— T
O188	4 ➤ I
O189	(I) × 3
O190	STO+ T
O191	5 ➤ I
O192	(I) × 4
O193	STO— T
O194	2
O195	STO× T
O196	2 ➤ I
O197	T ➤ (I)
O198	3 ➤ I
O199	(I) ➤ T
O200	4 ➤ I
O201	(I) × 4
O202	STO— T
O203	5 ➤ I
O204	(I) × 10
O205	STO+ T
O206	3 ➤ I
O207	8 × T ➤ (I)
O208	4 ➤ I
O209	(I) ➤ T
O210	5 ➤ I

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Line	Instruction
O211	(I) × 6
O212	STO– T
O213	4 ➤ I
O214	T × 32 ➤ (I)
O215	5 ➤ I
O216	COS(G)^6 × (I) × 128➤ T
O217	4 ➤ I
O218	COS(G)^4 × (I)
O219	STO+ T
O220	3 ➤ I
O221	COS(G)^2 × (I)
O222	STO+ T
O223	2 ➤ I
O224	RCL (I)
O225	STO+ T
O226	COS(G) × SIN(G)
O227	RCL× T
O228	→DEG
O229	RCL+ G
O230	STO F
O231	→RAD(ATAN((J × COS(O) + W × SIN(O)) ÷ COS(→DEG(U ÷ D)))) ÷ B
O232	→DEG
O233	RCL+ Y
O234	STO L
O235	(LN((1+SIN(F))÷(1–SIN(F)))–SQRT(E)×LN((1+SQRT(E)×SIN(F))÷(1–SQRT(E)×SIN(F))))÷2 ➤ Q
O236	SINH(B × Q + C) ➤ J
O237	COSH(B × Q + C) ➤ I
O238	→DEG (→RAD (L – Y) × B) ➤ H
O239	→HMS(ATAN((SIN(O) – J × COS(O) × SIN(–H))÷(I × COS(O) × COS(–H)) – M) ➤ G
O240	1 ➤ I //numeral 1 (one) stored in variable I//
O241	(I)× SQRT(1 – E × SIN(F)^2) × COS(→DEG(U ÷ D)) ÷ COS(F) ÷ COS(–H) ➤ K
O242	→HMS (F) ➤ F
O243	→HMS (L) ➤ L
****	Show results
O244	SF 10
O245	RESULTS
O246	PSE
O247	LATITUDE
O248	PSE
O249	VIEW F
O250	LONGITUDE
O251	PSE
O252	VIEW L
O253	GRID CONV

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Line	Instruction
O254	PSE
O255	VIEW G
O256	PT SCALE FACT
O257	PSE
O258	VIEW K
O259	0
O260	STO Z
O261	NEXT PT [0—1]
O262	PSE
O263	INPUT Z
O264	RCL Z
O265	x = 0?
O266	GTO O294 //Go to end//
O267	NEW ZONE [0—1]
O268	PSE
O269	0
O270	STO Z
O271	INPUT Z
O272	CF 10
O273	6 ➤ J
O274	(J) ➤ F
O275	7 ➤ J
O276	(J) ➤ L
O277	8 ➤ J
O278	TAN ((J)) ➤ M ‡
O279	9 ➤ J
O280	(J) ➤ K
O281	10 ➤ J
O282	(J) ➤ R
O283	11 ➤ J
O284	(J) ➤ S
O285	RCL Z
O286	SF 10
O287	x = 0?
O288	GTO O058 //Go to same zone//
O289	CF 10
O290	→HMS (F) ➤ F
O291	→HMS (L) ➤ L
O292	SF 10

Line	Instructions
O293	GTO O028 //Go to new zone//
O294	PROGRAM END
O295	PSE
****	Clear indirect memory
O296	11
O297	STO I
O298	0
O299	STO (I)
O300	1
O301	STO— I
O302	RCL I
O303	x ≥ 0?
O304	GTO O298
O305	FS? 1
O306	CF 10
O307	RTN

‡ Lines 143 and 279: the TAN function is used on the (J) indirect variable, then stored in M.

Notes

(1) The program should be run in RPN mode, as results in ALG mode are unknown.

(2) Latitudes and longitudes should be entered in HP notation, i.e., DDD.MMSS. The grid convergence and the output latitude and longitude are displayed in HP notation.

(3) The program may be used for any Oblique Mercator projection, if the appropriate parameters are known. Similarly, any ellipsoid may be used, if its a and e^2 parameters are known. Parameters for a wide range of ellipsoids, the single SPCS zone and the four Great Lakes zones are included at the end of this document.

(4) Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in North America. It is critical to enter the correct sign in the calculator when entering values.

(5) Lines with **** are comments only, and should not be entered into the calculator. They are there to make program entry a little easier. Similarly, text on a statement line that is within a pair of // marks is also a comment, to aid with understanding the code.

(6) This program is long and often appears to be a stream of meaningless commands and equations. This means that it may be more prone to errors when being entered. It is suggested that the program be entered using the given constants, tested (and the length and checksum checked), and when everything is satisfactory, the constants at the start of the program can be changed to those most suitable for the bulk of the expected work. They are set for Alaska Zone 1, as currently entered.

(7) The ➤ symbol near the end of an equation is the STO command. Simply key in STO and the variable name, and it will appear as written. STO+, STO—, STO× and STO÷ will not work in this context; you have to write them into the equation before the final STO operation.

(8) When entering equations, you must press the EQN key first. The various functions then appear with names and parentheses. Use the left and right cursor keys to navigate into and out of parentheses, as needed. Extra parentheses may be added as needed, as a pair, and one of them may be deleted to allow a single parenthesis to be placed.

(9) The use of equations, rather than direct instruction code, does slow the computation process a little, but makes the program a lot shorter (in terms of lines) and so possibly easier to enter.

(10) The program allows the user to run additional points after each is completed, by prompting. If another point is to be processed, the user also has the option to move to a new zone and ellipsoid, otherwise the previous ellipsoid and projection parameters are used. Respond 0 for 'NO' and 1 for 'YES' at the Z? prompt. If the user chooses to enter another point, the previous ellipsoid and zone specific data entered is displayed at the prompts if a new zone is chosen, but this is skipped if the same zone is requested.

(11) The program allows the user to select whether to convert from lat/long to E/N or vice versa. The reason for having it all in the one program is that there are a number of constants that are required for both processes, so it was better to program these once, rather than duplicating them across two programs.

(12) If your use of the program was for the Alaska Zone 1 in SPCS 1983 only, after getting it running correctly, you could insert a GTO O053 statement immediately after line O027. This would avoid having to confirm all the ellipsoid and zone parameters.

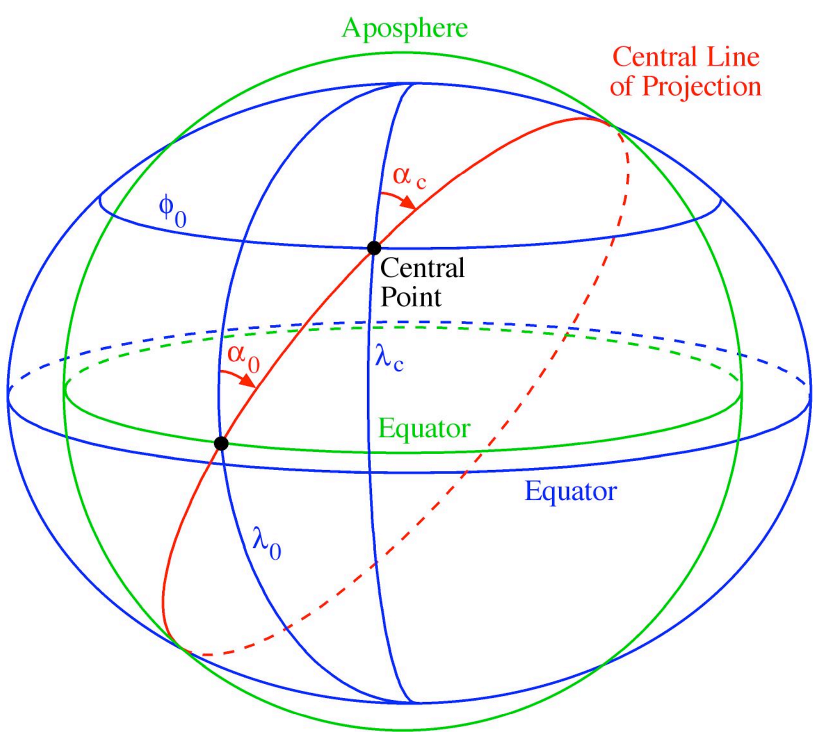
Theory

The modern implementations of the Oblique Mercator projection are largely based on Martin Hotine's work, and the projection is also known as the Hotine Oblique Mercator. It was in use before Hotine's work, but he was the one who formulated the equations used today, especially those for the US SPCS and the Great Lakes Zones. Hotine himself called it the "rectified skew orthomorphic" projection in his work on map projections.

Because the ellipsoid does not present a regular surface for the central line for a cylindrical projection, when it is other than the equator or a meridian, the central line of an Oblique Mercator projection produces some added complications in doing the projection. Hotine projected the ellipsoid onto an 'aposphere' (which could be a sphere of constant total curvature, but was actually non-spherical), and from there to a plane. A significant advantage in Hotine's method, as compared to previous methods using conformal spheres was that the solution equations were closed (i.e., didn't involve infinite series), albeit using hyperbolic functions. Variations in scale along the central line, i.e., the aposphere distance compared to the ellipsoidal distance, were also reduced significantly. The projection is conformal.

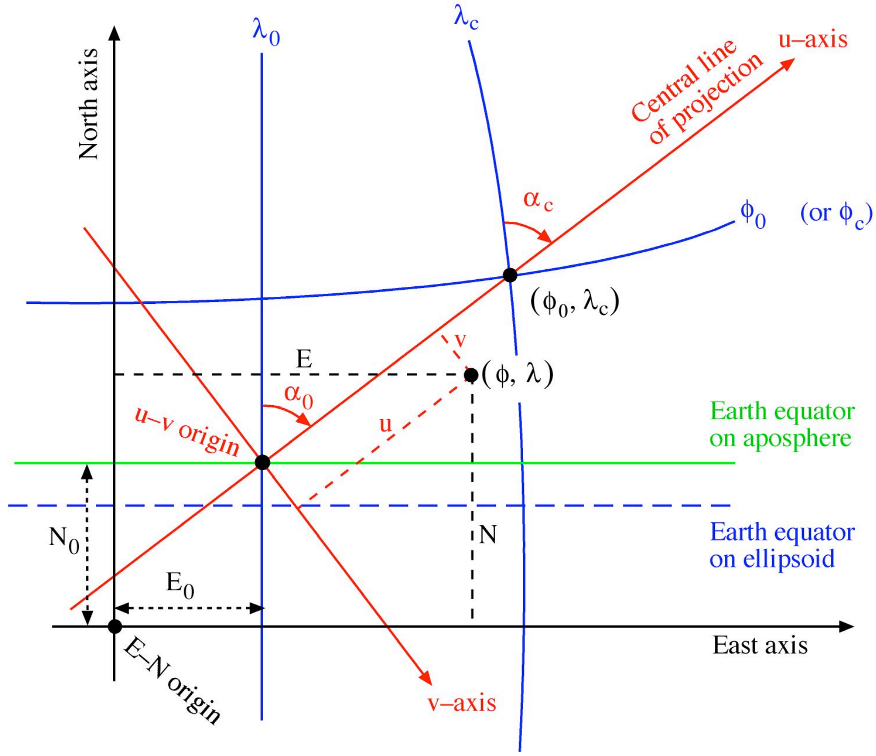
Unlike a Transverse Mercator projection, the scale varies slightly along the central line, and isolines of constant scale tend to be narrow ellipse-like figures around what is termed the central point of the projection. The central point is a convenient point on which to base the projection close to the area of interest.

Compared to a Transverse Mercator projection, Hotine's approach has the origin of the co-ordinate system at the point where the central line meets the equator of the aposphere. Determining where this point is in reality is not important, but it is important for the calculations, as the distance from that point along the central line to a point of interest (actually, its projection to the central line) is equivalent to the latitude on a Transverse Mercator projection, and this distance is used to determine what is effectively 'meridian convergence' on the aposphere, which in turn influences the co-ordinates, the scale factor (since this varies largely with distance from the central line), the grid convergence and the arc-to-chord corrections.



Because the projection process from ellipsoid to plane actually involves two projections, there are some additional complications. The nature of the aposphere is such that many of its equations involve hyperbolic functions, so the projection involves both trigonometric and hyperbolic functions, often in the one equation. Further, the conversion process is broken up into manageable pieces, so there are several equations and a number of intermediate terms.

The figure on the previous page shows the ellipsoid (in blue) and an aposphere (in green) that is fitted to the central line of the projection (in red). The central point is a point in the middle of the area to be mapped, while the true co-ordinate origin is where the central line crosses the aposphere's equation, which is where the meridian λ_0 may be projected from the ellipsoid onto the aposphere. The azimuth of the central line changes along the line, and it is designated α_c at the central point, and α_0 at the co-ordinate origin at λ_0.



The above figure shows a view of the 'local' area of an Oblique Mercator projection, considered in the mapping plane, and showing the various co-ordinate systems. The projection of the ellipsoid's equator onto the aposphere will generally not coincide with the equator of the aposphere. In this case, as is the situation with the five Oblique Mercator projections whose parameters are presented below, the projection is defined by a central point, (ϕ_0, λ_C), and an orientation, α_C, of the central line at the central point.

The ellipsoid's latitude and longitude graticule is shown in blue, while the equator of the aposphere is shown in green. The projection's co-ordinate system, u–v, with origin where the central line intersects the equator of the aposphere, at longitude λ_0, is shown in red. The projection from the aposphere to a plane (the analogue is a cylinder wrapped around the aposphere, with the line of contact being the central line) allows the cartesian u–v co-ordinate system to be used. This system is then rotated and shifted into the E–N system, shown in black, so that Grid North is in some convenient direction, and so that co-ordinates are positive and increase in the approximate east and north directions.

In the five projections discussed in this document, the assumption is that Grid North matches geodetic north at the central point of the projection.

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Conversion Formulae that Apply to Both Conversions

The required information to undertake a conversion is as follows.

From the ellipsoid definition, the usual parameters, a and e^2, are needed. These are supplied in a table later in this document. A series of four values, F_0, F_2, F_4 and F_6, are required for the conversion from E/N to lat/long, which are all functions of e^2. These are calculated by the program, to allow for the use of different ellipsoids, although they are constant for any given ellipsoid.

The required parameters relating to the specific projections are:

E_0	the false easting, or the easting offset;
N_0	the false northing, or the northing offset;
ϕ_0 or ϕ_c	the geodetic latitude of the local origin, positive north;
λ_c	the geodetic longitude of the local origin, positive east;
α_c	the azimuth of the positive skew axis (the u-axis) at the local origin; and
k_c	the point scale factor at the local origin.

The subscript C refers to parameters concerning the central point of the projection.

Several of the following intermediate values are constants for a given projection, and are used in both forward and backward conversions. In the program, they are computed first, before the calculator goes on to the specific conversion. Note that e in the equations below is always the square root of e^2, the ellipsoid parameter, unless otherwise noted. ln(x) is the natural logarithm of x. The values marked with asterisks are used in the conversion equations, while the others are intermediate results only.

* $$B = \sqrt{\left(1 + \frac{e^2 \cos^4 \phi_C}{1 - e^2}\right)}$$

$$W_C = \sqrt{\left(1 - e^2 \sin^2 \phi_C\right)} \qquad \left[= \frac{\nu}{a} \right]$$

$$A = \frac{a\,B\,\sqrt{\left(1 - e^2\right)}}{W_C^2}$$

$$Q_C = \frac{1}{2}\left(\ln\left(\frac{1 + \sin\phi_C}{1 - \sin\phi_C}\right) - e \ln\left(\frac{1 + e\sin\phi_C}{1 - e\sin\phi_C}\right)\right)$$

where Q_C is the isometric latitude of the local origin

* $$C = \operatorname{arccosh}\left(\frac{B\sqrt{\left(1 - e^2\right)}}{W_C \cos\phi_C}\right) - BQ_C$$

where $\operatorname{arccosh} x = \cosh^{-1} x = \ln\left(x + \sqrt{\left(x^2 - 1\right)}\right)$

$$* \qquad D = \frac{k_C\ A}{B} = \frac{k_C\ a\sqrt{\left(1 - e^2\right)}}{1 - e^2 \sin^2 \phi_C} = k_C \sqrt{\rho_C \nu_C} = r_C$$

$$\sin\alpha_0 = \frac{a\ \sin\alpha_C\ \cos\phi_C}{A\ W_C}$$

where α_0 is the azimuth of the positive skew axis (the u–axis) at the equator of the aposphere. This value is needed in later calculations.

$$* \qquad I = \frac{k_C\ A}{a}$$

$$* \qquad \lambda_0 = \lambda_C - \frac{1}{B}\ \arcsin\left(\frac{\sin\alpha_0\ \sinh\left(BQ_C + C\right)}{\cos\alpha_0}\right)$$

where λ_0 is longitude of the true origin, i.e., where the central line (the u–axis) crosses the equator of the aposphere. As an aside:

ν = radius of curvature in the prime vertical at ϕ; i.e. $\nu = \dfrac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$

$\rho = \dfrac{a\ (1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}}$ = radius of curvature in the meridian at ϕ

Usually, R is used to designate the geometric mean of the principal radii of curvature at a point, i.e., $R = \sqrt{\rho\nu}$, which can be used as a 'generic' radius at that point, such as for calculating scale factors and arc-to-chord corrections. Where there is a scale factor applied to the central line, this radius is scaled and is designated r, i.e., $r = k_0\sqrt{\rho\nu}$. This R is not the same as the R in the equations below.

For manual calculation, it is often handy to compute the value of sin α_0 and cos α_0 and store these, as they are used several times. However, owing to the limited direct storage in the calculator, they are calculated each time they are used. The equations below reflect this approach.

Converting Latitude and Longitude to Eastings and Northings

Given the latitude and longitude of the points to be converted, ϕ, λ, the conversion proceeds as follows:

$$H = \left(\lambda_0 - \lambda\right)B$$

noting that the $(\lambda_0 - \lambda)$ part should be in radians before multiplying by B, but H should be in whatever units are being used for the computation.

Oblique Mercator Co-ordinates to/from Latitude/Longitude

$$Q = \frac{1}{2}\left(\ln\left(\frac{1+\sin\phi}{1-\sin\phi}\right) - e\ln\left(\frac{1+e\sin\phi}{1-e\sin\phi}\right)\right)$$

where Q is the isometric latitude of the point to be converted.

$$J = \sinh(B\,Q + C)$$

$$K = \cosh(B\,Q + C)$$

$$u = D\ \arctan\left(\frac{J\cos\alpha_0 - \sin\alpha_0 \sin H}{\cos H}\right)$$

$$v = \frac{D}{2}\ \ln\left(\frac{K - J\ \sin\alpha_0 - \cos\alpha_0 \sin H}{K + J\ \sin\alpha_0 + \cos\alpha_0 \sin H}\right)$$

where u and v are the converted point's co-ordinates in the u–v system based on the central line of the projection. These are then converted to the East/North co-ordinate system.

$$E = u \sin \alpha_c + v \cos \alpha_c + E_0$$

$$N = u \cos \alpha_c - v \sin \alpha_c + N_0$$

The grid convergence, γ, and point scale factor, k, at the converted point are calculated as follows:

$$\gamma = \arctan\left(\frac{\sin\alpha_0 - J\cos\alpha_0 \sin H}{K\cos\alpha_0 \cos H}\right) - \alpha_C$$

$$k = \frac{I\ \sqrt{\left(1-e^2\sin^2\phi\right)}\ \ \cos\left(\frac{u}{D}\right)}{\cos\phi\ \ \cos H}$$

noting that $\frac{u}{D}$ will be in radians, and should be converted to the angular units used for computation.

Converting Eastings and Northings to Latitude and Longitude

Given the E and N co-ordinates of the point to be converted, the co-ordinates are first converted to the u–v co-ordinate system, as follows:

$$u = (E - E_0) \sin \alpha_c + (N - N_0) \cos \alpha_c$$

$$v = (E - E_0) \cos \alpha_c - (N - N_0) \sin \alpha_c$$

$$R = \sinh\left(\frac{v}{D}\right)$$

$$S = \cosh\left(\frac{v}{D}\right)$$

$$T = \sin\left(\frac{u}{D}\right)$$

noting that $\frac{u}{D}$ will be in radians, and should be converted to the angular units used for computation.

$$Q = \left(\frac{1}{B}\right)\left(\left(\frac{1}{2}\right)\ln\left(\frac{S - R\sin\alpha_0 + T\cos\alpha_0}{S + R\sin\alpha_0 - T\cos\alpha_0}\right) - C\right)$$

$$\chi = 2\arctan\left(\frac{e^Q - 1}{e^Q + 1}\right)$$

where χ is the conformal latitude, and e = 2.718 281 828 … (the base of natural logarithms) in this case only, and where e is raised to the power of Q, the isometric latitude, in both numerator and denominator. The geodetic latitude is obtained as follows:

$$\phi = \chi + \sin\chi \cos\chi \left(F_0 + F_2\cos^2\chi + F_4\cos^4\chi + F_6\cos^6\chi\right)$$

and the geodetic longitude is obtained thus:

$$\lambda = \lambda_0 + \frac{1}{B}\arctan\left(\frac{R\cos\alpha_0 + T\sin\alpha_0}{\cos\left(\frac{u}{D}\right)}\right)$$

noting that the value returned from the arctan function must be in radians before being divided by B. The result is then converted to match the angular units of the stored λ_0.

The F values for the latitude calculation are determined as follows, and are constants for any given ellipsoid:

$$c_2 = \frac{e^2}{2} + \frac{5\,e^4}{24} + \frac{e^6}{12} + \frac{13\,e^8}{360}$$

$$c_4 = \frac{7\,e^4}{48} + \frac{29\,e^6}{240} + \frac{811\,e^8}{11520}$$

$$c_6 = \frac{7\,e^6}{120} + \frac{81\,e^8}{1120}$$

$$c_8 = \frac{4279\ e^8}{161280}$$

$$F_0 = 2\left(c_2 - 2c_4 + 3c_6 - 4c_8\right) \quad = 0.006\ 686\ 920\ 927$$

$$F_2 = 8\left(c_4 - 4c_6 + 10c_8\right) \quad = 5.201\ 458\ 439 \times 10^{-5}$$

$$F_4 = 32\left(c_6 - 6c_8\right) \quad = 5.544\ 299\ 179 \times 10^{-7}$$

$$F_6 = 128c_8 \quad = 6.820\ 452\ 543 \times 10^{-9}$$

The values given for the F constants are those for the GRS-80 ellipsoid, which is the basis of NAD83. Note that WGS84 may be slightly different (depending upon which variant of WGS84 is used), as will the Clarke 1866 ellipsoid underlying the 1927 SPCS.

Some Constants

Alaska Zone 1 SPCS

The various zone constants for Alaska Zone 1 (Zone 5001) in the SPCS are as follows:

SPCS 1983	SPCS 1927
a = 6,378,137 m	a = 20 925 832.2 ft = 6378206.4 m
e^2 = 0.006 694 3800	e^2 = 0.006 768 66
λ_C = 133° 40' W	λ_C = 133° 40' W
ϕ_0 = 57° 00' N	ϕ_0 = 57° 00' N
B = 1.000 296 461 404	B = 1.000 299 772 823
C = 0.004 426 833 926	C = 0.004 475 992 640
D = 6 386 186.732 53	D = 20 952 558.763 17
sin α_0 = —0.327 012 955 4	sin α_0 = —0.327 015 517 245
cos α_0 = 0.945 019 855 3	cos α_0 = 0.945 018 698 847
I = 1.001 558 917 662	I = 1.001 577 360 008
λ_0 = —101°.513 839 560	λ_0 = —101°.514 031 447 351
E_0 = 5 000 000.000 m	E_0 = 16 404 166.694 ft ≈ 5 000 000.000 m
N_0 = —5 000 000.000 m	N_0 = —16 404 166.705 ft ≈ —5 000 000.00 m
α_0 = —19°.087 573 975 3	α_0 = —19°.087 729 294 986
α_C = —36°.869 897 645 85	α_C = —36°.869 897 645 8
tan α_C = —0.75	tan α_C = —0.75
k_C = 0.999 900	k_C = 0.999 900

Oblique Mercator Co-ordinates to/from Latitude/Longitude

The E_0 and N_0 values for SPCS 1927 were supposed to be ± 5 000 000.000 m, but the small differences are needed to match the results from the NGS on-line program. The Oblique Mercator Projection was adapted for use in Alaska SPCS Zone 1 about 1960 by Erwin Schmid of the US Coast & Geodetic Survey. It was set up with meters as the basis, but was used with US Survey feet.

Great Lakes Zones

The US Lake Survey adopted the Oblique Mercator projection about 1970 to map the Great Lakes area in four zones. These projections are all based on the Clarke 1866 ellipsoid and 1927 datum. As the US Lake Survey was incorporated into NOAA in 1976, these projections appear never to have been updated to the NAD83 datum, and so remain in the 1927 system. The various offset parameters were specified in meters, but are also provided here in US Survey feet. Note that these projections are not part of the SPCS.

Zone 1 (Erie, Ontario, St. Lawrence R.)	**Zone 2** (Huron)
a = 20 925 832.2 ft (= 6378206.4 m)	a = 20 925 832.2 ft (= 6378206.4 m)
e^2 = 0.006 768 66	e^2 = 0.006 768 66
λ_C = 78° 00' W	λ_C = 82° 30' W
ϕ_0 = 44° 00' N	ϕ_0 = 43° 00' N
B = 1.000 911 928 254	B = 1.000 974 362 623
C = 0.002 707 815 336	C = 0.002 572 976 258
D = 20 921 139.557 69	D = 20 918 663.748 41
sin α_0 = 0.594 513 704 0	sin α_0 = —0.119 339 996 589
cos α_0 = 0.804 085 477 9	cos α_0 = 0.992 853 445 990
I = 1.000 687 472 585	I = 1.000 631 464 133
λ_0 = —123°.417 048 710	λ_0 = —76°.082 341 472
E_0 = —3 950 000.000 m	E_0 = 1 200 000.000 m
—12 959 291.666 ft	3 937 000.000 ft
N_0 = —3 430 000.000 m	N_0 = —3 500 000.000 m
—11 253 258.333 ft	—11 482 916.667 ft
α_0 = 36°.477 972 922 2	α_0 = —6°.854 013 449 720
α_C = 55° 40' 00"	α_C = 350° 37' 00"
tan α_C =	tan α_C =
k_C = 0.999 900	k_C = 0.999 900

These values were computing using the foot value for a for the ellipsoid, as were those below. Note that several of the constants for Zone 3 are the same as those for Zone 1 because the latitude of the central point is the same in both cases (i.e., 44° 00' N), and the relevant constants are functions of this latitude only.

Zone 3 (Michigan)	**Zone 4** (Superior, Lake of the Woods)
a = 20 925 832.2 ft (= 6378206.4 m)	a = 20 925 832.2 ft (= 6378206.4 m)
e^2 = 0.006 768 66	e^2 = 0.006 768 66
λ_C = 87° 00' W	λ_C = 85° 50' 00".256 W
ϕ_0 = 44° 00' N	ϕ_0 = 47° 12' 21".554 N
B = 1.000 911 928 254	B = 1.000 725 565 272
C = 0.002 707 815 336	C = 0.003 144 939 509
D = 20 921 139.557 69	D = 20 929 086.317 44
sin α_0 = 0.186 336 887 6	sin α_0 = —0.654 587 036 760
cos α_0 = 0.982 485 910 5	cos α_0 = 0.755 986 647 571
I = 1.000 687 472 585	I = 1.000 881 185 296
λ_0 = —97°.524 992 069	λ_0 = —19°.850 809 611
E_0 = —1 000 000.000 m	E_0 = 9 000 000.000 m
—3 280 833.333 ft	29 527 500.000 ft
N_0 = —4 300 000.000 m	N_0 = —1 600 000.000 m
—14 107 583.333 ft	—5 249 333.333 ft
α_0 = 10°.739 085 841 4	α_0 = —40°.888 344 308 997
α_C = 15° 00' 00"	α_C = 285° 41' 42".593
tan α_C =	tan α_C =
k_C = 0.999 900	k_C = 0.999 900

Sample Computations

Example 1

Using the SPCS 1983 (a = 6,378,137 m, e^2 = 0.006 694 3800), the following results are obtained.

Alaska Zone 1, 5001, λ_0 = –133° 40', ϕ_0 = 57° 00', k_0 = 0.999 900,
E_0 = 5,000,000.000 m, N_0 = —5,000,000.000 m, tan α_C = —0.75

Easting (E) = 774,398.097 m Northing (N) = 715,316.601 m

Latitude = 58° 15' 25".000 Longitude = –134° 25' 15".000

Grid Convergence (γ) = –0° 37' 56.13" Point Scale Factor (k) = 0.999 929 06

These values agree with the NGS software conversion package, going both ways.

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Example 2

Using the SPCS 1927 ($a = 20925832.2$ ft, $e^2 = 0.006\ 768\ 66$), the following results are obtained.

Alaska Zone 1, 5001 1927, $\lambda_0 = -133° 40'$, $\phi_0 = 57° 00'$, $k_0 = 0.999\ 900$,
$E_0 = 16\ 404\ 166.694$ m, $N_0 = -16\ 404\ 166.705$ m, $\tan \alpha_C = -0.75$

Easting (E) = 2,540,366.483 ft Northing (N) = 2,347,240.712 ft

Latitude = 58° 15' 25".000 Longitude = –134° 25' 15".000

Grid Convergence (γ) = –0° 37' 56.13" Point Scale Factor (k) = 0.999 929 06

These values agree with the NGS software conversion package, going both ways, to within about a millimeter.

Example 3

Using the SPCS 1927 ($a = 20925832.2$ ft, $e^2 = 0.006\ 768\ 66$), the following results are obtained.

Great Lakes Zone 1, $\lambda_C = -78° 00'$, $\phi_0 = 44° 00'$, $k_0 = 0.999\ 900$,
$E_0 = -12{,}959.291.666$ ft, $N_0 = -11{,}253{,}258.333$ ft
$\tan \alpha_C = 1.646\ 114\ 728\ 394$ $\alpha_C =$

Easting (E) = 5,336.479.217 ft Northing (N) = 1,739,716.979 ft

Latitude = 45° 55' 35" Longitude = —76° 48' 20"

Grid Convergence (γ) = +0° 49' 36".41 Point Scale Factor (k) = 1.000 092 19

These values work through the program both ways, but there are no external sites for checking, other than an Excel spreadsheet developed for the purpose.

Example 4

Using the SPCS 1927 ($a = 20925832.2$ ft, $e^2 = 0.006\ 768\ 66$), the following results are obtained.

Great Lakes Zone 4, $\lambda_0 = -88° 50' 00''.256$, $\phi_0 = 47° 12' 21.554$, $k_0 = 0.999\ 900$,
$E_0 = 29{,}527.500.000$ ft, $N_0 = -5{,}249{,}333.333$ ft
$\tan \alpha_C = -3.626\ 951\ 197\ 402$ $\alpha_C = 285° 41' 42''.593$

Easting (E) = 2,323,564.650 ft Northing (N) = 2,531,813.212 ft

Latitude = 48° 15' 25 Longitude = —90° 25' 15"

Grid Convergence (γ) = –1° 09' 58.91" Point Scale Factor (k) = 0.999 983 54

These values work through the program both ways, but there are no external sites for checking, other than an Excel spreadsheet developed for the purpose.

Because there is no apparent external check for the Great Lakes Zones, it is a wise move to convert the values back to the originals, to check if they agree. This will allow you to check for any data entry errors. After a conversion from lat/long to E/N, the Easting and Northing values remain in the T and N registers, respectively, and so by doing the Next Point, same zone, entering E–N, the values can be checked without any additional data entry. However, the F and L registers are used for making sure the same zone information is available, and these overwrite the latitude and longitude values, after a E/N to lat/long conversion. So the latitude and longitude will need to be re-entered for a backwards check.

Running the Program

Press XEQ O , then the ENTER key, to start the program. The calculator briefly displays OBLIQUE MERCTR, then briefly shows CHECK—ENTER A. This is "Point A," discussed below. The program then stops and displays the prompt for entering the semi-major axis value, while displaying the current default value:

A?
6,378,137.0000 (This is for GRS80/NAD83)

If you are happy with this value for the semi-major axis of the ellipsoid, press R/S to continue. Otherwise. key in a different value (for a different ellipsoid, e.g., 20925832.2 ft for Clarke 1866) and press R/S to continue. (This discussion will use the data from Example 1, in the Sample Computations section, above.)

The calculator briefly displays CHECK—ENTER E. The program then stops and displays the prompt for entering the eccentricity of the ellipsoid, e:

E?
0.00669438 (This is for GRS80/NAD83)

If this value for the eccentricity is correct, press R/S to continue. Otherwise, key in a different value (for a different ellipsoid, e.g., 0.006 768 66 for Clarke 1866) and press R/S to continue.

The calculator briefly displays CHECK—ENTER K. The program then stops and displays the prompt for entering the scale factor at the central point (λ_C), which is k_C:

K?
0.9999000 (This is for Alaska SPCS Zone 1 and all four Great Lakes Zones)

If this value for the scale factor is satisfactory, press R/S to continue. If you want to change it, key in the correct value and press R/S.

The calculator briefly displays CHK—NTR LONG C. The program then stops and displays the prompt for entering the longitude of the central point of the projection, λ_C. Note that in the western hemisphere, this will be a negative value, and should be in HP notation (DDD.MMSS).

L?
—133.400000 (This is for Alaska Zone 1)

If this is the correct central point longitude, press R/S to continue, if this is not correct, key in the correct value, in HP notation, then press R/S to continue.

The calculator briefly displays CHK—NTR LAT 0. The program then stops and displays the prompt for entering the latitude of the projection's central point, ϕ_0 or ϕ_C. The value should be entered in HP notation.

F?
57.000000 (This is for Alaska Zone 1)

If this is the correct latitude, press R/S to continue. If you want a different value, key in that value and press R/S to continue.

The calculator briefly displays CHK—NTR E 0. The program then stops and prompts for entry of the false easting value, or the easting offset, denoted E_0.

R?
5,000,000.0000 (This is for Alaska Zone 1)

If this is the correct value, press R/S to continue. If a different value is desired, key in the value and press R/S.

The calculator briefly displays CHK—NTR N 0. The program then stops and prompts for the false northing value, or the northing offset.

S?
—5,000,000.0000 (This is for Alaska Zone 1)

If this is the correct value, press R/S to continue. If a different value is required, key in the value and press R/S.

The calculator briefly displays CHK—NTR TAN AC. The program then stops and prompts for the tangent of the azimuth of the projection's central line, at the central point, tan α_C.

M?
—0.750000000 (This is for Alaska Zone 1)

If this is the correct value, press R/S to continue. If a different value is required, key it in and press R/S.

This is "Point B," discussed below.

The calculator briefly displays RUNNING, then briefly displays X—Y IN [0—1] to ask whether you wish to enter latitude and longitude, or easting and northing, for conversion. The calculator stops and displays the answer prompt:

Z?
0.000000

If you want to convert latitude and longitude to cartesian co-ordinates, just press R/S for the default response (i.e., 0 for no). If you want to convert easting and northing to latitude and longitude, key in 1 and press R/S. The latter is discussed a little further along this section of the document.

Case 0: Latitude and Longitude to Easting and Northing

If it is desired to convert from latitude and longitude, and R/S alone was pressed, the calculator displays RUNNING for a short while, as it calculated the general parameters for either conversion, the briefly displays ENTER PT LAT, then stops and displays the prompt to enter the latitude of the point to be converted:

F?
57.00000

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Note that the prompt will always display the ϕ_0 value. Key in the latitude of the point, in HP notation (DDD.MMSSsss), in this case 58.1525 and press R/S. The calculator then briefly displays ENTER PT LONG, then stops and prompts for entering the longitude of the point to be converted.

L?
—133.6666667

Note that the prompt always display the longitude λ_C, but in decimal degrees. Key in the correct longitude in HP notation (DDD.MMSSsss), in this case —134.2515 (remember it's a negative number!) and press R/S. The calculator now displays RUNNING for a while, as it does the conversion. The calculator briefly displays RESULTS, then briefly displays EASTING, then stops to display the computing easting value of the converted point:

T=
774,398.096730

Press R/S and the calculator briefly displays NORTHING, then stops and displays the northing of the converted point:

N=
715,316.600660

Press R/S and the calculator briefly displays GRID CONV, then stops and displays the grid convergence at the point, in HP notation:

G=
—0.37561234

which is the grid convergence value of —0° 37' 56".12. Press R/S and the calculator briefly displays PT SCALE FACT, then stops and displays the point scale factor at the converted point:

K=
0.99992906

Press R/S, and the calculator briefly displays NEXT PT [0—1], then stops and displays the prompt asking for whether to process another point. If you are finished, press R/S (the default response is 0 or no) and the calculator displays PROGRAM END, then RUNNING briefly while it clears the indirectly addressed memory used during the program run and resets Flag 10.

Z?
0.000000

(If you get an error about a non-existent memory register, this will be because there was no register 11 created, as this is only needed in the cartesian to lat/long conversion. Press the C (ON/OFF) key to cancel the error message and end the program, but the program will not reset Flag 10 to its former value.)

If you want to process another point, key in 1 and press R/S. The calculator briefly displays NEW ZONE [0—1], then briefly shows RUNNING, then prompts whether you want to use a different zone. If you want to use the same zone, just press R/S (the default response is 0 or no), and the program returns to Point B, above.

Z?
0.000000

If you want to use a different zone, key in 1 and press R/S. The calculator returns to Point A, above, which is almost at the start of the program.

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Case1: Easting and Northing to Latitude and Longitude

If you selected to convert cartesian co-ordinates (E/N) into geographical co-ordinates (lat/long), the calculator displays RUNNING for a short while, then briefly displays ENTER EASTING, then stops and prompts for the easting value of the point to be converted:

T?
774,398.0967

(Note that this value may be very different, depending upon previous work in the calculator.) Key in the easting of the point to be converted, in this case 774,398.0967, and press R/S. The calculator briefly displays ENTER NORTHING, then stops and prompts for the northing co-ordinate of the point to be converted:

N?
715,316.6007

(This will usually be the last northing used.) Key in the northing of the point to be converted, in this case 715,316.6007 and press R/S. The calculator displays RUNNING for a while, the briefly displays RESULTS, then briefly displays LATITUDE, and then stops and displays the latitude of the converted point:

F=
58.15250000

This is the latitude of the point in HP notation, in this case 58° 15' 25". Press R/S and the calculator briefly displays LONGITUDE, then stops and displays the longitude of the converted point:

L=
—134.25150000

This is the longitude of the point in HP notation, in this case —134° 25' 15". Press R/S and the calculator briefly displays GRID CONV, then stops and displays the grid convergence value:

G=
—0.375613

This is the grid convergence at the converted point in HP notation, in this case —0° 37' 56".13. Press R/S and the calculator briefly displays PT SCALE FACT, then stops and displays the point scale factor of the converted point:

K=
0.99992906

Press R/S, and the calculator briefly displays NEXT PT [0—1], then stops and displays the prompt asking for whether to process another point. If you are finished, press R/S (the default response is 0 or no) and the calculator displays PROGRAM END, then RUNNING briefly while it clears the indirectly addressed memory used during the program run, and resets Flag 10.

Z?
0.000000

If you want to process another point, key in 1 and press R/S. The calculator briefly displays NEW ZONE [0—1], then briefly shows RUNNING, then prompts whether you want to use a different zone. If you want to use the same zone, just press R/S (the default response is 0 or no), and the program returns to Point B, above.

Z?
0.000000

If you want to use a different zone, key in 1 and press R/S. The calculator returns to Point A, above, which is almost at the start of the program.

Main Storage Registers Used

A	Semi-major axis of the ellipsoid being used, a
B	Internal computational value, specific to the projection and zone
C	Internal computational value, specific to the projection and zone
D	Internal computational value, specific to the projection and zone
E	Eccentricity of the ellipsoid, e^2
F	ϕ_c, latitude of the center point of the projection; then the latitude to be converted, ϕ.
G	γ, the grid convergence of the point being converted, and internal computational values
H	longitudinal difference, multiplied by B, i.e., $(\lambda - \lambda_0)$ B
I	Internal computational value
J	Internal computational value
K	k_c, the point scale factor at the central point; then the point scale factor at the converted point, k
L	λ_c, the longitude of the central point of the projection; then the longitude to be converted, λ.
M	α_c, the angle of the central line of the projection from the meridian, at the central point (initially the tangent of that angle)
N	Converted or entered northing co-ordinate
O	α_0, the angle of the central line of the projection with the equator
P	Internal computational value
Q	Internal computational value
R	E_0, the easting offset, or false easting, for the projection, and internal computational values
S	N_0, the northing offset, or false northing, for the projection, and internal computational values
T	Converted or entered easting co-ordinate, and internal computational values
U	Co-ordinate on the projection, prior to 'rectified' to an E/N grid; along the central line
V	Co-ordinate on the projection, prior to 'rectified' to an E/N grid; perpendicular to the central line
W	Internal computational value
X	Easting co-ordinate of point to be converted
Y	λ_0, the point where the central line of the projection crosses the equator, and where the projection effectively starts.
Z	used for getting responses to questions about running more points

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Indirect Storage Registers Used

1	I, an internal value
2	c_2, then F_0
3	c_4, then F_2
4	c_6, then F_4
5	c_8, then F_6
6	ϕ_0
7	λ_0
8	α_C
9	k_C
10	E_0
11	N_0

Statistical Registers: not used

Labels Used

Label **O** Length = 2969 Checksum = 8719

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computations to check proper operation after entry.

Flags Used

Flags 1 and 10 are used by this program. Flag 10 is set for part of this program, so that equations can be shown as prompts. Elsewhere, it is cleared, so that equations can be evaluated. Flag 1 is used to record the setting of Flag 10 before the program begins. At the end of the program, Flag 10 is reset to its original value, based on the value in Flag 1.

Basic Parameters for the Computations

State Plane Co-ordinate System (SPCS) 1983

The only instance of the Oblique Mercator Projection in the SPCS is Zone 1 of Alaska (Zone 5001), which covers the Alaskan panhandle. The ellipsoid used in GRS80 on the NAD83 datum, and the basic parameters for this Zone are as follows. Various constants for this zone may be found above.

a = 6,378,137 m	e^2 = 0.006 694 3800
λ_C = 133° 40' W	ϕ_0 = 57° 00' N
E_0 = 5 000 000.000 m	N_0 = —5 000 000.000 m
α_C = —36°.869 897 645 85	tan α_C = —0.75
k_C = 0.999 900 = 1: 10 000 reduction	

Oblique Mercator Co-ordinates to/from Latitude/Longitude

State Plane Co-ordinate System (SPCS) 1927

The only instance of the Oblique Mercator Projection in the SPCS is Zone 1 of Alaska (Zone 5001), which covers the Alaskan panhandle. The ellipsoid used is the Clarke 1866 on the NAD27 datum, and the basic parameters for this Zone are as follows. Various constants for this zone may be found above.

$a = 20\,925\,832.2$ ft $= 6378206.4$ m $e^2 = 0.006\,768\,66$

$\lambda_C = 133° \, 40'$ W $\phi_0 = 57° \, 00'$ N

$E_0 = 16\,404\,166.694$ ft $\approx 5\,000\,000.000$ m

$N_0 = -16\,404\,166.705$ ft $\approx -5\,000\,000.00$ m

$\alpha_C = -36°.869\,897\,645\,85$ $\tan \alpha_C = -0.75$

$k_C = 0.999\,900900 = 1{:}\,10\,000$ reduction

The E_0 and N_0 values for SPCS 1927 were supposed to be ± 5 000 000.000 m, commonly used with US Survey feet to match the rest of the 1927 system, but the small differences are needed to match the results from the NGS on-line program. The Oblique Mercator Projection was adapted for use in Alaska SPCS Zone 1 about 1960 by Erwin Schmid of the US Coast & Geodetic Survey.

Great Lakes Zones

The US Lake Survey adopted the Oblique Mercator projection about 1970 to map the Great Lakes area in four zones. These projections are all based on the Clarke 1866 ellipsoid and 1927 datum. As the US Lake Survey was incorporated into NOAA in 1976, these projections appear never to have been updated to the NAD83 datum, and so remain in the 1927 system. The various offset parameters were specified in meters, but are also provided here in US Survey feet. Note that these projections are not part of the SPCS.

Zone 1 (Erie, Ontario, St. Lawrence R.)	**Zone 2** (Huron)
$a = 20\,925\,832.2$ ft (= 6378206.4 m)	$a = 20\,925\,832.2$ ft (= 6378206.4 m)
$e^2 = 0.006\,768\,66$	$e^2 = 0.006\,768\,66$
$\lambda_C = 78° \, 00'$ W	$\lambda_C = 82° \, 30'$ W
$\phi_0 = 44° \, 00'$ N	$\phi_0 = 43° \, 00'$ N
$E_0 = -3\,950\,000.000$ m	$E_0 = 1\,200\,000.000$ m
$-12\,959\,291.666$ ft	$3\,937\,000.000$ ft
$N_0 = -3\,430\,000.000$ m	$N_0 = -3\,500\,000.000$ m
$-11\,253\,258.333$ ft	$-11\,482\,916.667$ ft
$\alpha_C = 55° \, 40' \, 00''$	$\alpha_C = 350° \, 37' \, 00''$
$\tan \alpha_C = 1.464\,114\,728\,394$	$\tan \alpha_C =$
$k_C = 0.999\,900$	$k_C = 0.999\,900$

Oblique Mercator Co-ordinates to/from Latitude/Longitude

Zone 3 (Michigan)

a = 20 925 832.2 ft (= 6378206.4 m)

e^2 = 0.006 768 66

λ_C = 87° 00' W

ϕ_0 = 44° 00' N

E_0 = —1 000 000.000 m
—3 280 833.333 ft

N_0 = —4 300 000.000 m
—14 107 583.333 ft

α_C = 15° 00' 00"

tan α_C =

k_C = 0.999 900

Zone 4 (Superior, Lake of the Woods)

a = 20 925 832.2 ft (= 6378206.4 m)

e^2 = 0.006 768 66

λ_C = 85° 50' 00".256 W

ϕ_0 = 47° 12' 21".554 N

E_0 = 9 000 000.000 m
29 527 500.000 ft

N_0 = —1 600 000.000 m
—5 249 333.333 ft

α_C = 285° 41' 42".593

tan α_C =

k_C = 0.999 900

Ellipsoids

There are a range of ellipsoids in common or former use. The table below has the a and e^2 values for a number of common (and less common) ellipsoids.

Ellipsoid	a Semi-major Axis	e^2 Eccentricity
GRS80–WGS94–NAD83	6378137 m	0.006 694 38
Clarke 1866 (NAD27)	6378206.4 m	0.006 768 66
Clarke 1866 (NAD27)	20925832.2 ft	0.006 768 66
ANS (Australian)	6378160 m	0.006 694 541 855
Airy 1830	6377563.4 m	0.006 670 54
Bessel 1841	6377397.16 m	0.006 674 372
Clarke 1880	6378249.15 m	0.006 803 511
Everest 1830	6377276.35 m	0.006 637 847
Fischer 1960 (Mercury)	6378166 m	0.006 693 422
Fischer 1968	6378150 m	0.006 693 422
Hough 1956	6378270 m	0.006 722 67
International	6378388 m	0.006 722 67
Krassovsky 1940	6378245 m	0.006 693 422
South American 1960	6378160 m	0.006 694 542
GRS 1967	6378160 m	0.006 694 605
GRS 1975	6378140 m	0.006 694 385
WGS 60	6378165 m	0.006 693 422
WGS 66	6378145 m	0.006 694 542
WGS 72	6378135 m	0.006 694 317 778
WGS 84	6378137 m	0.006 694 38

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