Mathematics 1

Roots of a Quadratic Equation

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Line	Instruction	User Instructions
Q001	LBL Q	► LBL Q
Q002	CLSTK	CLEAR 5
Q003	FS? 10	► FLAGS 3 .0
Q004	GTO Q008	
Q005	SF 1	S I FLAGS 1 1
Q006	SF 10	FLAGS 1 .0
Q007	GTO Q009	
Q008	CF 1	FLAGS 2 1
Q009	QUADRATIC ROOT	(Key in using EQN RCL Q RCL U, etc.,)
Q010	PSE	► PSE
Q011	ENTER A	(Key in using EQN RCL E RCL N, etc.,)
Q012	PSE	PSE PSE
Q013	INPUT A	S INPUT A
Q014	ENTER B	(Key in using EQN RCL E RCL N, etc.,)
Q015	PSE	▶ PSE
Q016	INPUT B	S INPUT B
Q017	ENTER C	(Key in using EQN RCL E RCL N, etc.)
Q018	PSE	PSE PSE
Q019	INPUT C	S INPUT C
Q020	RCL B	
Q021		$r x^2$
Q022	RCL A	
Q023	RCL× C	
Q024	4	
Q025	×	
Q026	_	
Q027	STO D	STO D
Q028	<i>x</i> < 0?	x?0 3 (<)
Q029	GTO Q054	[to 2 complex roots]
Q030	$\mathbf{x} = 0?$	x?0 6 (=)
Q031	GTO Q082	[to I real root]
Q032	RCL D	[2 real roots]
Q033	√X DCL D	_
Q034	RCL- B	_
Q035	RCL÷ A	_
Q036	2	_
Q037	÷	
Q038		
Q039	IST REAL ROOT	(Key in using EQN 1 RCL S RCL T SPACE, etc.)

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Line	Instruction	User Instructions
Q040	PSE	▶ PSE
Q041	VIEW P	SI VIEW P
Q042	RCL D	
Q043	$\sqrt{\mathbf{x}}$	
Q044	RCL+ B	
Q045	RCL÷ A	
Q046	2	
Q047	÷	
Q048	+/	
Q049	STO Q	STO Q
Q050	2ND REAL ROOT	(Key in using EQN 2 RCL N RCL D SPACE, etc.)
Q051	PSE	▶ PSE
Q052	VIEW Q	ST VIEW Q
Q053	GTO Q091	[to end part]
Q054	RCL D	[2 complex roots]
Q055	ABS	ABS
Q056	$\sqrt{\mathbf{x}}$	
Q057	i	
Q058	X	
Q059	RCL-B	
Q060	RCL÷ A	
Q061	2	
Q062	÷	
Q063	STO P	STO P
Q064	1ST CMPLX ROOT	(Key in using EQN 1 RCL S RCL T SPACE, etc.)
Q065	PSE	▶ PSE
Q066	VIEW P	✓ VIEW P
Q067	RCL D	
Q068	ABS	ABS
Q069	$\sqrt{\mathbf{x}}$	
Q070	+/	
Q071	i	
Q072	X	
Q073	RCL-B	
Q074	RCL÷ A	
Q075	2	
Q076	÷	
Q077	STO Q	► STO Q
Q078	2ND CMPLX ROOT	(Key in using EQN 2 RCL N RCL D SPACE, etc.)
Q079	PSE	PSE PSE
Q080	VIEW Q	VIEW Q
Q081	GTO Q091	[to end part]
Q082	RCL B	[1 real root]
Q083	+/	
Q084	RCL÷ A	
Q085	2	

Line	Instruction	User Instructions
Q086	÷	
Q087	STO P	► STO P
Q088	SINGLE ROOT	(Key in using EQN RCL S RCL I RCL N, etc.)
Q089	PSE	▶ PSE
Q090	VIEW P	ST VIEW P
O091	FS? 1	ST FLAGS 3 1
Q092	CF 10	← FLAGS 2 .0
Q093	STOP	R/S
Q094	RTN	ST RTN

Roots of a Quadratic Equation

Notes

- (1) A simple program to calculate the roots of a quadratic equation in standard form: $ax^2 + bx + c = 0.$
- (2) If there is a single real root for the equation, the value in storage location D is equal to zero, so after showing SINGLE ROOT, then the value in storage location P (the solution), the program ends.
- (3) If there are two real roots, the program displays 1ST REAL ROOT, displays the first solution (in storage location P), then displays 2ND REAL ROOT, then shows the second solution (in storage location Q).
- (4) If there are two complex roots, the program displays 1ST COMPLX ROOT, displays the first solution as a complex number (using the complex number display format selected), then displays 2ND COMPLX ROOT, and then the second solution as a complex number.
- (5) This program sets Flag 10, so that equations can be displayed as prompts. The program completes by returning Flag 10 to its setting when the program began.
- (6) This program may be called by another program. The roots are left on the stack after the program finishes, regardless of the approach used, as well as in storage locations P and Q. To simplify returning to the calling routine, omit the STOP at line Q093. It is there to allow the stand-alone use of the program to avoid automatically returning to some other program point.

Theory

The solution to a quadratic equation in the form $ax^2 + bx + c = 0$ is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $b^2 - 4ac$ is known as the discriminant. The discriminant, commonly termed Δ , is stored in storage location D.

If Δ is positive, the equation has two real roots. If $\Delta = 0$, the equation has a single, real root, i.e., it is a perfect square. If Δ is negative, the equation has two complex roots. The calculator solves for all of these situations.

Roots of a Quadratic Equation

Running the Program

Press XEQ Q, then press ENTER.

Program displays QUADRATIC ROOT briefly, then ENTER A briefly, then prompts: A?

Key in the value of a from the equation. Press R/S.

Program displays ENTER B briefly, then prompts: B?

Key in the value of b from the equation. Press R/S.

Program displays ENTER C briefly, then prompts: C?

Key in the value of c from the equation. Press R/S.

A. If there are two real roots, the program briefly displays 1ST REAL ROOT, then displays: P=

and the value of one of the roots of the equation.

Press R/S and the calculator briefly displays 2ND REAL ROOT, then displays: O=

and the value of the other root of the equation.

Press R/S and the calculator resets the various flags and the program ends.

B. If there are two complex roots, the calculator briefly displays 1ST CMPLX ROOT, then displays: P=

and the value of one of the roots, as a complex number. The value is displayed in the current calculator mode for displaying complex numbers. This may be set, and is explained below.

Press R/S and the calculator briefly displays 2ND CMPLX ROOT, then displays: O=

and the value of the other root, as a complex number.

Press R/S and the calculator resets the various flags and the program ends.

C. If there is a single root to the equation, i.e., the equation is a perfect square, the calculator briefly displays SINGLE ROOT, then displays:
P=

and the value of the single root.

Press R/S and the calculator resets the various flags and the program ends.

Complex Number Display

Complex numbers in the HP-35s can be displayed in two ways in RPN mode. These are the rectangular mode and polar mode. Which display mode is set depends upon the display setting. To see complex numbers in rectangular form, press \frown DISPLAY, then 9. The *x i y* mode shows numbers as: 25.3456 i -54.3210

HP-35s Calculator Program Roots of a Quadratic Equation

The first part is the real part of the number, and the *i* separates this from the imaginary part, which is the second part. So the above number would be read as 25.3456 - i54.3210.

The other display mode, polar, is selected by pressing \triangleleft DISPLAY, then selecting option 10 (scroll up or down to find it). This is shown as $r \theta a$, and shows first the distance, then the angle of this direction as an angle in decimal degrees counter-clockwise from the \Re (real) axis (horizontal axis to the right).

The same number as above would be shown as: $59.943060 \theta - 64.986744$

where the 59.943060 is the distance, and the -60.986744 is the angle in decimal degrees.

Complex numbers may be entered in either mode, but are displayed in the selected mode. The mode can be set or changed at any time, to suit the user's needs.

Sample Computations

1. Solve $4x^2 + x + 8 = 0$

There are two complex roots for this equation.

Solutions: 1st Complex Root P = -0.125 + i1.4086782nd Complex Root Q = -0.125 - i1.408678

2. Solve $4x^2 + 8x - 6 = 0$

There are two real roots for this equation.

Solutions: 1st Root P = 0.5811392nd Root Q = -2.581139

3. Solve $x^2 + 2x + 1 = 0$

There is just one root for this equation.

Solution: Single Root P = -1.00000

Storage Registers Used

- **A** Value of *a* in equation
- **B** Value of *b* in equation
- **C** Value of *c* in equation
- **D** Discriminant, Δ
- **P** Solution of the first root
- **Q** Solution of the second root

Statistical Registers: not used

Labels Used

Label **Q** Length = 390 Checksum = 4A3E

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.