HP-35s Calculator Program — Lambert 2

Convert Lambert Conformal Conic Projection Co-ordinates (SPCS) to Latitude and Longitude

Programmer: Dr. Bill Hazelton

Date: September, 2010. **Version**: 1.1 **Mnemonic**: **L** for **L**ambert to **L**atitude and **L**ongitude

Line	Instruction	Display	User Instructions
L001	LBL L		LBL L
L002	CLSTK		CLEAR 5
L003	FS? 10		FLAGS 3 .0
L004	GTO L008		
L005	SF 1		FLAGS 1 1
L006	SF 10		FLAGS 1 .0
L007	GTO L009		
L008	CF 1		FLAGS 2 1
L009	LCC 2 LAT–LONG		(Key in using EQN RCL L, RCL C, etc.)
L010	PSE		PSE
L011	CL x		CLEAR 1
L012	STO X		STO X
L013	STO Y		STO Y
L014	STO P		STO P
L015	STO Q		STO Q
L016	STO C		STO C
L017	STO D		STO D
L018	STO G		STO G
L019	STO H		STO H
L020	6378137		a value for ellipsoid (WGS84/NAD83)
L021	STO A		STO A
L022	0.00669438		e^2 value for ellipsoid (WGS84/NAD83)
L023	STO E		STO E
L024	CHECK–ENTER A		(Key in using EQN RCL C, RCL H, etc.)
L025	PSE		PSE
L026	INPUT A		INPUT A
L027	CHECK–ENTER E		(Key in using EQN RCL C, RCL H, etc.)
L028	PSE		PSE
L029	INPUT E		INPUT E
L030	RCL E		
L031	√x		
L032	STO O		STO O
L033	CHK—NTR LAT 0		(Key in using EQN RCL C, RCL H, etc.)
L034	PSE		PSE
L035	INPUT P		INPUT P
L036	CHK—NTR LONG 0		(Key in using EQN RCL C, RCL H, etc.)
L037	PSE		PSE
L038	INPUT Q		INPUT Q

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Line	Instruction
L039	STD PARALLEL 1
L040	PSE
L041	INPUT C
L042	STD PARALLEL 2
L043	PSE
L044	INPUT D
L045	CHK—NTR E 0
L046	PSE
L047	INPUT G
L048	CHK—NTR N 0
L049	PSE
L050	INPUT H
L051	RCL P
L052	HMS→
L053	STO P
L054	RCL Q
L055	HMS→
L056	STO Q
L057	RCL C
L058	HMS→
L059	STO C
L060	RCL D
L061	HMS→
L062	STO D
L063	ENTER EASTING
L064	PSE
L065	INPUT X
L066	ENTER NORTHING
L067	PSE
L068	INPUT Y
L069	RCL G
L070	STO— X
L071	RCL H
L072	STO— Y
****	Calculate m_1
L073	RCL C
L074	STO Z
L075	XEQ L224
L076	RCL Z
L077	STO R
****	Calculate m_2
L078	RCL D
L079	STO Z
L080	XEQ L224
L081	RCL Z

Line	Instruction
L082	STO S
****	Calculate t_1
L083	RCL C
L084	STO Z
L085	XEQ L237
L086	RCL Z
L087	STO U
****	Calculate t_2
L088	RCL D
L089	STO Z
L090	XEQ L237
L091	RCL Z
L092	STO V
****	Calculate t_0
L093	RCL P
L094	STO Z
L095	XEQ L237
L096	RCL Z
L097	STO T
****	Calculate n
L098	RCL R
L099	LN
L100	RCL S
L101	LN
L102	—
L103	RCL U
L104	LN
L105	RCL V
L106	LN
L107	—
L108	÷
L109	STO N
****	Calculate F
L110	RCL R
L111	RCL÷ N
L112	RCL U
L113	RCL N
L114	y^x
L115	÷
L116	STO J
****	Calculate r_0
L117	RCL A
L118	RCL× J
L119	RCL T
L120	RCL N

Line	Instruction
L121	y^x
L122	×
L123	STO M
****	Calculate θ
L124	RCL X
L125	RCL M
L126	RCL— Y
L127	÷
L128	ATAN
L129	STO W
****	Calculate r
L130	RCL X
L131	x^2
L132	RCL M
L133	RCL— Y
L134	x^2
L135	+
L136	$\sqrt{x}$
L137	STO B
****	Calculate λ
L138	RCL W
L139	RCL÷ N
L140	RCL+ Q
L141	→HMS
L142	STO L
****	Calculate ϕ
L143	90
L144	RCL B
L145	RCL÷ A
L146	RCL÷ J
L147	RCL N
L148	1/x
L149	y^x
L150	STO I
L151	ATAN
L152	2
L153	×
L154	—
L155	STO F
L156	FN= Z
L157	SOLVE F
L158	GTO L164
L159	CANNOT SOLVE
L160	PSE
L161	FOR LATITUDE

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Line	Instruction
L162	PSE
L163	GTO L219
****	Calculate scale factor
L164	1
L165	RCL F
L166	SIN
L167	x^2
L168	RCL× E
L169	—
L170	$\sqrt{x}$
L171	RCL A
L172	x < > y
L173	÷
L174	RCL F
L175	COS
L176	×
L177	RCL÷ N
L178	RCL÷ B
L179	1/x
L180	STO K
****	Calculate γ
L181	RCL W
L182	→HMS
L183	STO W
****	Show results
L184	SF 10
L185	RESULTS
L186	PSE
L187	LATITUDE
L188	PSE
L189	RCL F
L190	→HMS
L191	STO F
L192	VIEW F
L193	LONGITUDE
L194	PSE
L195	VIEW L
L196	GRID CONV
L197	PSE
L198	VIEW W
L199	PT SCALE FACT
L200	PSE
L201	VIEW K
****	Check for next pt.
L202	0
L203	STO Z

Line	Instruction
L204	NEXT PT [0—1]
L205	PSE
L206	INPUT Z
L207	RCL Z
L208	x = 0 ?
L209	GTO L219
L210	NEW ZONE [0—1]
L211	PSE
L212	0
L213	STO Z
L214	INPUT Z
L215	RCL Z
L216	x = 0 ?
L217	GTO L063
L218	GTO L024
****	End of program
L219	PROGRAM END
L220	PSE
L221	FS? 1
L222	CF 10
L223	RTN
****	Subroutines
****	Compute m
L224	RCL Z
L225	COS
L226	RCL Z
L227	SIN
L228	x^2
L229	RCL× E
L230	1
L231	x < > y
L232	—
L233	$\sqrt{x}$
L234	÷
L235	STO Z
L236	RTN
****	Compute t
L237	1
L238	RCL Z
L239	SIN
L240	RCL× O
L241	—
L242	RCL Z
L243	SIN
L244	RCL× O
L245	1

Line	Instruction
L246	+
L247	÷
L248	RCL O
L249	2
L250	÷
L251	y^x
L252	45
L253	RCL Z
L254	2
L255	÷
L256	—
L257	TAN
L258	x < > y
L259	÷
L260	STO Z
L261	RTN
****	*********
****	Calculate ϕ
Z001	LBL Z
Z002	1
Z003	RCL F
Z004	STO Z
Z005	SIN
Z006	RCL× O
Z007	—
Z008	RCL F
Z009	SIN
Z010	RCL× O
Z011	1
Z012	+
Z013	÷
Z014	RCL O
Z015	2
Z016	÷
Z017	y^x
Z018	RCL× I
Z019	ATAN
Z020	2
Z021	×
Z022	90
Z023	x < > y
Z024	—
Z025	STO F
Z026	RCL— Z
Z027	RTN

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

Notes

(1) The program should be run in RPN mode, as results in ALG mode are unknown.

(2) Latitudes and longitudes should be entered in HP notation, i.e., DDD.MMSS. The grid convergence is displayed in HP notation.

(3) The program may be used for any Lambert Conformal Conic projection, if the appropriate parameters are known. Similarly, any ellipsoid may be used, if its a and e^2 parameters are known. Parameters for a wide range of ellipsoids and all SPCS Lambert zones are included at the end of this document.

(4) Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in North America. It is critical to enter the correct sign in calculator when entering values.

(5) Lines with **** are comments only, and should not be entered into the calculator. They are there to make program entry a little easier.

(6) This program is long and often appears to be a stream of meaningless commands. This means that it may be more prone to errors when being entered. It is suggested that the program be entered using the given constants, tested (and the checksum checked), and when it is satisfactory, the values for the zone that are set to zero at the start of the program can be changed to those most suitable for the bulk of the expected work. See the **Localization** section at the end of the document.

(7) When working in SPCS 1927, there are some small differences in latitudes between this program and the NGS conversion program. This may be caused by a different method of computing the distance from the pole in days gone by (see the discussion in the **Theory** section). The differences are small (less than 0.25 inch in all places tested thus far) and appear to be larger the further the point is from the pole. The conversions in SPCS 1983 agree to 0.001 m, which is the finest value the NGS program provides.

(8) The scale factor is exactly 1.0000 when the point is on the standard parallels. It is less than 1.0000 between the standard parallels, and greater than 1.0000 outside the standard parallels.

Theory

Unlike the Transverse Mercator projection, where the forward and reverse co-ordinate conversions are pretty straight-forward, if long-winded, the solution to the conversion of grid co-ordinates to geographical co-ordinates on the Lambert Conformal Conic is a rather more complex affair, requiring an iterated solution of the latitude of the point.

Given a, e^2, ϕ_1, ϕ_2, ϕ_0, λ_0, E_0, N_0, and the x and y co-ordinates of the point to be converted (E and N or X and Y, with false origin removed), we can calculate n, F and r_0 from the USGS formulae given below. You substitute ϕ_0, ϕ_1, or ϕ_2 into the formulae for m and t to get the appropriate values (denoted by subscripts) required. The value of e in the formula for t is $\sqrt{e^2}$.

$$r_0 = a F t_0^n$$

$$n = \frac{\ln m_1 - \ln m_2}{\ln t_1 - \ln t_2} \qquad \text{(constant of the projection or cone)}$$

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$$F = \frac{m_1}{n\, t_1^{\,n}}$$

$$m = \frac{\cos\phi}{\sqrt{1 - e^2 \sin^2\phi}}$$

$$t = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\left(\frac{1 - e\ \sin\phi}{1 + e\ \sin\phi}\right)^{\frac{e}{2}}}$$

The following values may be computed using the co-ordinates (less the false origin values), as follows:

$$x = E - E_0$$

$$y = N - N_0$$

$$r = \sqrt{x^2 + (r_0 - y)^2}$$

$$\theta = \arctan\left(\frac{x}{r_0 - y}\right)$$

$$\lambda = \frac{\theta}{n} + \lambda_0$$

Then we have to undertake an iterative solution to the following equation, solving for ϕ with a rough estimate for ϕ, substituting that value back into the equation and solving for a better value of ϕ. When there is no significant change in ϕ, we can stop the process.

A good starting estimate for ϕ is $\phi = \frac{\pi}{2} - 2\arctan t$.

The formula that is being iterated is:

$$\phi = \frac{\pi}{2} - 2\arctan\left(t\left(\frac{1 - e\sin\phi}{1 + e\sin\phi}\right)^{e/2}\right)$$

where

$$t = \left(\frac{r}{a\,F}\right)^{1/n}$$

Using the following formulae, the other various required quantities can be calculated

$\gamma = \theta$ (grid convergence at the point)

$k = \dfrac{n\ r}{\nu\cos\phi}$ (scale factor at the latitude ϕ)

where ν = the radius of the ellipsoid at the parallel of latitude ϕ

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Note that n, F and r_0 are constants for a particular map or SPCS zone and only need to be computed once.

The iterative solution lends itself to the SOLVE capability in the calculator, and allows this part to be programmed economically and fairly simply, albeit at the cost of an additional label. When it was attempted to use SOLVE on an equation, the calculator ran out of memory, as using equations in SOLVE is memory hungry.

To avoid iterations, you can use the following series expansion (note that Snyder gives some clues for faster implementation of this expansion on page 19 of his book). I have not tried this out, so cannot comment on its effectiveness, but it may be better suited to being run in the HP-33S calculator (no loops required!).

$$\begin{aligned}\phi = \chi &+ \left(\frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360} + \dots\right) \sin 2\chi \\ &+ \left(\frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^8}{11520} + \dots\right) \sin 4\chi \\ &+ \left(\frac{7e^6}{120} + \frac{81e^8}{1120} + \dots\right) \sin 6\chi + \left(\frac{4279e^8}{161280} + \dots\right) \sin 8\chi\end{aligned}$$

where

$$\chi = \frac{\pi}{2} - 2 \arctan t$$

If you are using the tables for SPCS 27 and these were derived from geocentric latitudes, you can use the same formulae, except that t for calculations leading to n, F and r_0 (formulae given above) is calculated using:

$$t = \tan\left(\frac{\pi}{2} - \frac{\phi_g}{2}\right)$$

where

$$\phi_g = \frac{\pi}{2} - 2 \arctan t$$

and the t in this equation is derived from $\quad t = \left(\frac{r}{a\,F}\right)^{1/n}$

The 1927 solutions, based on slightly different calculation of ϕ and hence t, lead to slightly different results in the calculation of the final latitude from the co-ordinates. The differences are apparent in the example calculations given. You may want to check the differences across the region being converted and apply an average correction to the values computed by the calculator, or do the 1927 conversions on-line.

(Note that the SPCS 27 has co-ordinates in US Survey feet, and uses the Clarke 1866 ellipsoid. The SPCS 83 has co-ordinates in meters, and uses the GRS80 spheroid, which effectively is the same as WGS-84. Some states have either the US Survey foot or the International foot as alternative distance units; check which one is in use in the state you are working in at any particular time. Note that there is a datum shift between the two systems (1927 and 1983) as well, and that you cannot really do a direct linear shift between them.

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As you can see from the tables below, most of the SPCS Lambert zones adopt ϕ_0 as having a Y or N value of zero. It is chosen so as to be well south of the limits of the zone. For most of the Lambert zones, the central meridian gets a value of 2,000,000 feet (SPCS 27), and a range of meter values for SPCS 83. See the tables below for exact data.

With the two standard parallels for the zone, it doesn't matter which is used for ϕ_1 and which for ϕ_2. Provided that the values for t_1, t_2, m_1 and m_2 are calculated and applied consistently, n, F and r_0 turn out the same either way. For convenience in the northern hemisphere, the southern parallel is used as ϕ_1, but this is not necessary for proper operation of the program.

Sample Computations

Example 1

Using the SPCS 1983 (a = 6,378,137 m, e^2 = 0.006 694 3800), the following results are obtained.

Ohio North Zone, 3401: True Origin: ϕ_0 = 39° 40', λ_0 = –82° 30';
Standard Parallels: ϕ_1 = 40° 26', ϕ_2 = 41° 42';
False Origin: E_0 = 600,000.000 m, N_0 = 0.000 m.

Easting (E) = 542,668.995 m Northing (N) = 47,416.966 m

Latitude = 40° 05' 30" Longitude = –83° 10' 20"

Grid Convergence (γ) = –0° 26' 29.82" Point Scale Factor (k) = 1.000 082 97

Example 2

Using the SPCS 1927 (a = 20,925,832.2 ft, e^2 = 0.006 768 66), the following results are obtained.

Ohio North Zone, 3401: True Origin: ϕ_0 = 39° 40', λ_0 = –82° 30';
Standard Parallels: ϕ_1 = 40° 26', ϕ_2 = 41° 42';
False Origin: E_0 = 2,000,000.000 ft, N_0 = 0.000 ft.

Easting (E) = 1,811,901.577 ft Northing (N) = 155,564.399 ft

Latitude = 40° 05' 30".000 0 Longitude = –83° 10' 20".000 0

Grid Convergence (γ) = –0° 26' 29.82" Point Scale Factor (k) = 1.000 082 97

Note: the NGS conversion program gave the same results, except for the latitude, which it gave as 40° 05' 30".000 06, a difference of 0.006 ft (about 0.07 inches or 1.8 mm), and –83° 10' 20".000 01 for the longitude, a difference of 0.000 75 ft (about 0.009 inches or 0.3 mm). Testing other points in this zone indicate a consistent difference of about this amount. This may be because of the different method of computing the distances from the pole (r and r_0) in earlier computations of the zones. Note that the NGS conversion from State Plane to geographical co-ordinates does not provide the grid convergence and scale factor. To get this, you will need to use a different converter.

Example 3

Using the SPCS 1927 (a = 20,925,832.2 ft, e^2 = 0.006 768 66), the following results are obtained.

California III Zone (0403), SPCS 1927, ϕ_0 = 36° 30', λ_0 = –120° 30';
ϕ_1 = 37° 04', ϕ_2 = 38° 26';
E_0 = 2,000,000.000 ft, N_0 = 0.000 ft.

Easting (E) = 2,216,169.136 ft Northing (N) = 338,664.251 ft

Latitude = 37° 25' 40".000 0 Longitude = –119° 45' 20".000 0

Grid Convergence (γ) = 0° 27' 20.8" Point Scale Factor (k) = 0.999 945 01

Note: the NGS conversion program gave the same results, except for the latitude, which it gave as 37° 25' 40".000 12, a difference of 0.012 ft (about 0.14 inches or 3.6 mm). Testing other points in this zone indicate a consistent difference of about this amount. This may be because of the different method of computing the distances from the pole (r and r_0) in earlier computations of the zones. Note that the NGS conversion from State Plane to geographical co-ordinates does not provide the grid convergence and scale factor. To get this, you will need to use a different converter.

Example 4

Using the SPCS 1983 (a = 6,378,137 m, e^2 = 0.006 694 3800), the following results are obtained.

California III Zone (0403), SPCS 1983, ϕ_0 = 36° 30', λ_0 = –120° 30';
ϕ_1 = 37° 04', ϕ_2 = 38° 26';
E_0 = 2,000,000.000 m, N_0 = 500,000.000 m.

Easting (E) = 2,065,886.861 m Northing (N) = 603,227.485 m

Latitude = 37° 25' 40" Longitude = –119° 45' 20"

Grid Convergence (γ) = 0° 27' 20.8" Point Scale Factor (k) = 0.999 945 01

Running the Program

Press XEQ L, then press ENTER to start the program. The calculator briefly displays LCC 2 LAT-LONG, then briefly shows CHECK—ENTER A. The program then stops and displays the prompt for entering the semi-major axis value, while displaying the current default value:

A?
6,378,137.0000 (This is for GRS80/WGS84/NAD83)

If you are happy with this value for the semi-major axis of the ellipsoid, press R/S to continue. Otherwise. Key in a different value (for a different ellipsoid) and press R/S to continue.

The calculator briefly displays CHECK—ENTER E. The program then stops and displays the prompt for entering the eccentricity of the ellipsoid, e:

E?
0.00669438 (This is for GRS80/WGS84/NAD83)

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If this value for the eccentricity is correct, press R/S to continue. Otherwise, key in a different value (for a different ellipsoid) and press R/S to continue.

The calculator briefly displays CHK—NTR LAT 0. The program then stops and displays the prompt for entering the origin latitude for the co-ordinate, ϕ_0:

P?
0.0000

Key in the correct latitude in HP notation (DDD.MMSS), and press R/S to continue. In this case, key in 39.40 for Ohio North.

The calculator briefly displays CHK—NTR LONG 0. The program then stops and displays the prompt for entering the longitude of the central meridian of the projection, λ_0. Note that in the western hemisphere, this will be a negative value, and should be in HP notation (DDD.MMSS).

Q?
0.0000

Key in the correct longitude, in HP notation and remembering the sign, then press R/S to continue. In this case, key in –82.30 for Ohio North

The calculator briefly displays STD PARALLEL 1. The program then stops and displays the prompt for entering the latitude of one of the standard parallels for the projection, ϕ_1. The value should be entered in HP notation.

C?
0.000000

Key in the correct value and press R/S to continue. In this case, key in 40.26 and press R/S to continue.

The calculator briefly displays STD PARALLEL 2. The program then stops and displays the prompt for entering the latitude of the other standard parallel for the projection, ϕ_2. The value should be entered in HP notation.

D?
0.000000

Key in the correct value and press R/S to continue. In this case, key in 41.42 and press R/S to continue.

The calculator briefly displays CHK—NTR E 0. The program then stops and prompts for the false easting value, or the easting offset. This is the value of the easting at the central meridian (λ_0), denoted E_0.

G?
0.0000

Key in the correct value, and press R/S to continue. In this case, key in 600000.0 and press R/S.

The calculator briefly displays CHK—NTR N 0. The program then stops and prompts for entry of the false northing value, or the northing offset. This is the value of the northing co-ordinate at ϕ_0, λ_0.

H?
0.0000

If this is the correct value (for some zones, it is zero), press R/S to continue. If a different value is desired, key in the value and press R/S. In this case, just press R/S. This is the N_0 value for Ohio North.

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The calculator briefly displays ENTER EASTING. The program stops and displays the prompt for entering the easting of the point to be converted. This should be entered in HP notation.

X?
0.0000

Key in the easting of the point and press R/S to continue. In this case, key in 542668.995 and press R/S.

The calculator briefly displays ENTER NORTHING. The program then stops and displays the prompt for entering the northing of the point to be converted. This should be entered in HP notation.

Y?
0.0000

Key in the longitude of the point in HP notation and press R/S to continue. In this case, key in 47416.966 and press R/S/

The program displays RUNNING for a short while, then SOLVING for a while, then displays RESULTS briefly, followed by LATITUDE briefly. The program then stops and displays the latitude value of the point. In this case, the calculator displays:

F=
40.05300000

This is the latitude of the point, in HP notation (DDD.MMSSsss, i.e., 40° 05' 30".00 N). Press R/S to continue. The calculator briefly displays LONGITUDE, then stops and displays the northing value of the point. In this case, the calculator displays:

L=
—83.10200000

This is the longitude of the point, in HP notation (DDD.MMSSsss, i.e., 83° 10' 20".00 W). Press R/S to continue. The calculator briefly displays GRID CONV, then stops and displays the grid convergence value in HP notation. In this case, the calculator displays:

K=
—0.26298198

This is the grid convergence in HP notation, and is —0° 26' 29".82 in more conventional notation. Press R/S to continue. The calculator briefly displays PT SCALE FACT, then stops and displays the point scale factor of the point on the Lambert Conformal Conic projection. In this case, the calculator displays:

W=
1.00008297

This is the point scale factor. Press R/S to continue.

You now have the choice of running one or more additional points. The calculator briefly displays NEXT PT [0—1], then stops and displays the prompt for answering questions:

Z?
0.0000

If you want to quit the program, just press R/S, the calculator briefly displays PROGRAM END and then comes to an end, returning to the point whence it was called, or to normal operations. If you want to enter more points, key in 1 and press R/S. In this case, the calculator then prompts to see if you want to use the same parameters. The calculator briefly displays NEW ZONE [0—1], then stops at the question prompt:

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Z?
0.0000

If you want to go to a new zone, key in 1 and press R/S, and the calculator will take you to the point where you can change any of the values (Point A above), starting with the ellipsoid parameters. If you want to work in the same zone already entered, just press R/S, and the program will take you to "Point B" and prompt for the latitude of the point to be converted, and continue from there. You can go around the program as many times as necessary.

Reference

SNYDER, J.P., 1987. *Map Projections—A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington: US Government Printing Office.

Storage Registers Used

A Semi-major axis of the ellipsoid being used, a
B r, distance from the pole to the point
C ϕ_1, one of the two standard parallels of the projection
D ϕ_2, one of the two standard parallels of the projection
E Eccentricity of the ellipsoid, e^2
F ϕ, latitude of the point being converted
G E_0, the false easting or easting offset
H N_0, the false northing, or northing offset, at ϕ_0, λ_0
I Temporary variable t in the latitude solution.
J F, an internal computed value
K point scale factor, k
L λ, longitude of the point being converted
M r_0, distance from the pole to ϕ_0
N n, the constant of the projection or cone
O e, the square root of the eccentricity of the ellipsoid.
P ϕ_0, the latitude of the co-ordinate origin on the projection
Q λ_0, the central meridian of the projection
R m_1, an internal computed value
S m_2, an internal computed value
T t_0, an internal computed value
U t_1, an internal computed value
V t_2, an internal computed value
W θ, the angle between the line from the pole to the point, and the central meridian, also the grid convergence
X Easting co-ordinate of converted point
Y Northing co-ordinate of converted point
Z Response variable for checking if another point, as well as passing values to subroutines.

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

Statistical Registers: not used

Labels Used

Label **L**	Length = 1065	Checksum = 7096
Label **Z**	Length = 87	Checksum = 1E33

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Flags Used

Flags 1 and 10 are used by this program. Flag 10 is set for this program, so that equations can be shown as prompts. Flag 1 is used to record the setting of Flag 10 before the program begins. At the end of the program, Flag 10 is reset to its original value, based on the value in Flag 1.

Parameters for the Computations

Ellipsoids

There are a range of ellipsoids in common or former use. The table below has the a and e^2 values for a number of common (and less common) ellipsoids.

Ellipsoid	a Semi-major Axis	e^2 Eccentricity
GRS80–WGS94–NAD83	6378137 m	0.006 694 38
Clarke 1866 (NAD27)	6378206.4 m	0.006 768 66
Clarke 1866 (NAD27)	20925832.2 ft	0.006 768 66
ANS (Australian)	6378160 m	0.006 694 541 855
Airy 1830	6377563.4 m	0.006 670 54
Bessel 1841	6377397.16 m	0.006 674 372
Clarke 1880	6378249.15 m	0.006 803 511
Everest 1830	6377276.35 m	0.006 637 847
Fischer 1960 (Mercury)	6378166 m	0.006 693 422
Fischer 1968	6378150 m	0.006 693 422
Hough 1956	6378270 m	0.006 722 67
International	6378388 m	0.006 722 67
Krassovsky 1940	6378245 m	0.006 693 422
South American 1960	6378160 m	0.006 694 542
GRS 1967	6378160 m	0.006 694 605
GRS 1975	6378140 m	0.006 694 385
WGS 60	6378165 m	0.006 693 422
WGS 66	6378145 m	0.006 694 542
WGS 72	6378135 m	0.006 694 317 778
WGS 84	6378137 m	0.006 694 38

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

State Plane Co-ordinate System (SPCS) 1983

Several US states use the Lambert Conformal Conic projection for SPCS 1983. The various parameters for each zone in the 1983 system are given in the table below. Use these parameters with the program, together with the GRS80/WGS84/NAD83 ellipsoid parameters, in meters.

	Standard Parallels		**Origin Longitude**	**Origin Latitude**	**False East**	**False North**
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (m)	N_0 (m)
Alaska						
Zone 10	51° 50'	53° 50'	176° 00'	51° 00'	1,000,000.00	0.00
Arkansas						
North	34° 56'	36° 14'	92° 00'	34° 20'	400,000.00	0.00
South	33° 18'	34° 46'	92° 00'	32° 40'	400,000.00	400,000.00
California						
I	40° 00'	41° 40'	122° 00'	39° 20'	2,000,000.00	500,000.00
II	38° 20'	39° 50'	122° 00'	37° 40'	2,000,000.00	500,000.00
III	37° 04'	38° 26'	120° 30'	36° 30'	2,000,000.00	500,000.00
IV	36° 00'	37° 15'	119° 00'	35° 20'	2,000,000.00	500,000.00
V	34° 02'	35° 28'	118° 00'	33° 30'	2,000,000.00	500,000.00
VI	32° 47'	33° 53'	116° 15'	32° 10'	2,000,000.00	500,000.00
Colorado						
North	39° 43'	40° 47'	105° 30'	39° 20'	914,401.83	304,800.61
Central	38° 27'	39° 45'	105° 30'	37° 50'	914,401.83	304,800.61
South	37° 14'	38° 26'	105° 30'	36° 40'	914,401.83	304,800.61
Connecticut	41° 12'	41° 52'	72° 45'	40° 50'	304800.61	152400.30
Florida						
North	29° 35'	30° 45'	84° 30'	29° 00'	600000.00	0.00
Iowa						
North	42° 04'	43° 16'	93° 30'	41° 30'	1500000.00	1000000.00
South	40° 37'	41° 47'	93° 30'	40° 00'	500000.00	0.00
Kansas						
North	38° 43'	39° 47'	98° 00'	38° 20'	400000.00	0.00
South	37° 16'	38° 34'	98° 30'	36° 40'	400000.00	400000.00

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

	Standard Parallels		Origin Longitude	Origin Latitude	False East	False North
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (m)	N_0 (m)
Kentucky						
North	37° 58'	38° 58'	84° 15'	37° 30'	500,000.00	0.00
South	36° 44'	37° 56'	85° 45'	36° 20'	500,000.00	500,000.00
Louisiana						
North	31° 10'	32° 40'	92° 30'	30° 30'	1,00,0000.00	0.00
South	29° 18'	30° 42'	91° 20'	28° 30'	1,000,000.00	0.00
Offshore	26° 10'	27° 50'	91° 20'	25° 30'	1,000,000.00	0.00
Maryland	38° 18'	39° 27'	77° 00'	37° 40'	400,000.00	0.00
Massachusetts						
Mainland	41° 43'	42° 41'	71° 30'	41° 00'	200,000.00	750,000.00
Island	41° 17'	41° 29'	70° 30'	41° 00'	500,000.00	0.00
Michigan						
North	45° 29'	47° 05'	87° 00'	44° 47'	8,000,000.00	0.00
Central	44° 11'	45° 42'	84° 22'	43° 19'	6,000,000.00	0.00
South	42° 06'	43° 40'	84° 22'	41° 30'	4,000,000.00	0.00
Minnesota						
North	47° 02'	48° 38'	93° 06'	46° 30'	800,000.00	100000.00
Central	45° 37'	47° 03'	94° 15'	45° 00'	800,000.00	100,000.00
South	43° 47'	45° 13'	94° 00'	43° 00'	800,000.00	100,000.00
Montana	45° 00'	49° 00'	109° 30'	44° 15'	600,000.00	0.00
Nebraska	40° 00'	43° 00'	100° 00'	39° 50'	500,000.00	0.00
New York						
Long Island	40° 40'	41° 02'	74° 00'	40° 10'	300,000.00	0.00
North Carolina	34° 20'	36° 10'	79° 00'	33° 45'	609,601.22	0.00
North Dakota						
North	47° 26'	48° 44'	100° 30'	47° 00'	600,000.00	0.00
South	46° 11'	47° 29'	100° 30'	45° 40'	600,000.00	0.00
Ohio						
North	40° 26'	41° 42'	82° 30'	39° 40'	600,000.00	0.00
South	38° 44'	40° 02'	82° 30'	38° 00'	600,000.00	0.00

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

	Standard Parallels		Origin Longitude	Origin Latitude	False East	False North
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (m)	N_0 (m)
Oklahoma						
North	35° 34'	36° 46'	98° 00'	35° 00'	600,000.00	0.00
South	33° 56'	35° 14'	98° 00'	33° 20'	600,000.00	0.00
Oregon						
North	44° 20'	46° 00'	120° 30'	43° 40'	2,500,000.00	0.00
South	42° 20'	44° 00'	120° 30'	41° 40'	1,500,000.00	0.00
Pennsylvania						
North	40° 53'	41° 57'	77° 45'	40° 10'	600,000.00	0.00
South	39° 56'	40° 58'	77° 45'	39° 20'	600,000.00	0.00
Puerto Rico and Virgin Islands						
1	18° 02'	18° 26'	66° 26'	17° 50'	200,000.00	200,000.00
2 (St. Croix)	18° 02'	18° 26'	66° 26'	17° 50'		
Samoa	–14° 16'	–14° 16'	170° 00'			
South Carolina	32° 30'	34° 50'	81° 00'	31° 50'	609,600.00	0.00
South Dakota						
North	44° 25'	45° 41'	100° 00'	43° 50'	600,000.00	0.00
South	42° 50'	44° 24'	100° 20'	42° 20'	600,000.00	0.00
Tennessee	35° 15'	36° 25'	86° 00'	34° 20'	600,000.00	0.00
Texas						
North	34° 39'	36° 11'	101° 30'	34° 00'	200,000.00	1,000,000.00
North central	32° 08'	33° 58'	98° 30'	31° 40'	600,000.00	2,000,000.00
Central	30° 07'	31° 53'	100° 20'	29° 40'	700,000.00	3,000,000.00
South central	28° 23'	30° 17'	99° 00'	27° 50'	600,000.00	4,000,000.00
South	26° 10'	27° 50'	98° 30'	25° 40'	300,000.00	5,000,000.00
Utah						
North	40° 43'	41° 47'	111° 30'	40° 20'	500,000.00	1,000,000.00
Central	39° 01'	40° 39'	111° 30'	38° 20'	500,000.00	2,000,000.00
South	37° 13'	38° 21'	111° 30'	36° 40'	500,000.00	3,000,000.00
Virginia						
North	38° 02'	39° 12'	78° 30'	37° 40'	3,500,000.00	2,000,000.00
South	36° 46'	37° 58'	78° 3'0	36° 20'	3,500,000.00	1,000,000.00

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

	Standard Parallels		Origin Longitude	Origin Latitude	False East	False North
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (m)	N_0 (m)
Washington						
North	47° 30'	48° 44'	120° 50'	47° 00'	500,000.00	0.00
South	45° 50'	47° 20'	120° 30'	45° 20'	500,000.00	0.00
West Virginia						
North	39° 00'	40° 15'	79° 30'	38° 30'	600,000.00	0.00
South	37° 29'	38° 53'	81° 00'	37° 00'	600,000.00	0.00
Wisconsin						
North	45° 34'	46° 46'	90° 00'	45° 10'	600,000.00	0.00
Central	44° 15'	45° 30'	90° 00'	43° 50'	600,000.00	0.00
South	42° 44'	44° 04'	90° 00'	42° 00'	600,000.00	0.00

State Plane Co-ordinate System (SPCS) 1927

Several US states used the Lambert Conformal Conic projection for SPCS 1927. The various parameters for each zone in the 1927 system are given in the table below. Use these parameters with the program, together with the Clarke 1866 ellipsoid in feet.

	Standard Parallels		Origin Longitude	Origin Latitude	False Easting	False Northing
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (ft.)	N_0 (ft.)
Alaska						
Zone 10	51° 50'	53° 50'	176° 00'	51° 00'	3000000.00	0.00
Arkansas						
North	34° 56'	36° 14'	92° 00'	34° 20'	2000000.00	0.00
South	33° 18'	34° 46'	92° 00'	32° 40'	2000000.00	0.00
California						
I	40° 00'	41° 40'	122° 00'	39° 20'	2000000.00	0.00
II	38° 20'	39° 50'	122° 00'	37° 40'	2000000.00	0.00
III	37° 04'	38° 26'	120° 30'	36° 30'	2000000.00	0.00
IV	36° 00'	37° 15'	119° 00'	35° 20'	2000000.00	0.00
V	34° 02'	35° 28'	118° 00'	33° 30'	2000000.00	0.00
VI	32° 47'	33° 53'	116° 15'	32° 10'	2000000.00	0.00
VII	33° 52'	34° 25"	118° 20'	34° 08'	4186692.58	4160926.74

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

	Standard Parallels		Origin Longitude	Origin Latitude	False Easting	False Northing
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (ft.)	N_0 (ft.)
Colorado						
North	39° 43'	40° 47'	105° 30'	39° 20'	2000000.00	0.00
Central	38° 27'	39° 45'	105° 30'	37° 50'	2000000.00	0.00
South	37° 14'	38° 26'	105° 30'	36° 40'	2000000.00	0.00
Connecticut	41° 12'	41° 52'	72° 45'	40° 50'	600000.00	0.00
Florida						
North	29° 35'	30° 45'	84° 30'	29° 00'	2000000.00	0.00
Iowa						
North	42° 04'	43° 16'	93° 30'	41° 30'	2000000.00	0.00
South	40° 37'	41° 47'	93° 30'	40° 00'	2000000.00	0.00
Kansas						
North	38° 43'	39° 47'	98° 00'	38° 20'	2000000.00	0.00
South	37° 16'	38° 34'	98° 30'	36° 40'	2000000.00	0.00
Kentucky						
North	37° 58'	38° 58'	84° 15'	37° 30'	2000000.00	0.00
South	36° 44'	37° 56'	85° 45'	36° 20'	2000000.00	0.00
Louisiana						
North	31° 10'	32° 40'	92° 30'	30° 40'	2000000.00	0.00
South	29° 18'	30° 42'	91° 20'	28° 40'	2000000.00	0.00
Offshore	26° 10'	27° 50'	91° 20'	25° 40'	2000000.00	0.00
Maryland	38° 18'	39° 27'	77° 00'	37° 50'	800000.00	0.00
Massachusetts						
Mainland	41° 43'	42° 41'	71° 30'	41° 00'	600000.00	0.00
Island	41° 17'	41° 29'	70° 30'	41° 00'	200000.00	0.00
Michigan (current)						
North	45° 29'	47° 05'	87° 00'	44° 47'	2000000.00	0.00
Central	44° 11'	45° 42'	84° 20'	43° 19'	2000000.00	0.00
South	42° 06'	43° 40'	84° 20'	41° 30'	2000000.00	0.00
Minnesota						
North	47° 02'	48° 38'	93° 06'	46° 30'	2000000.00	0.00
Central	45° 37'	47° 03'	94° 15'	45° 00'	2000000.00	0.00
South	43° 47'	45° 13'	94° 00'	43° 00'	2000000.00	0.00

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

	Standard Parallels		Origin Longitude	Origin Latitude	False Easting	False Northing
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (ft.)	N_0 (ft.)
Montana						
North	47° 51'	48° 43'	109° 30'	47° 00'	2000000.00	0.00
Central	46° 27'	47° 53'	109° 30'	45° 50'	2000000.00	0.00
South	44° 52'	46° 24'	109° 30'	44° 00'	2000000.00	0.00
Nebraska						
North	41° 51'	42° 49'	100° 00'	41° 20'	2000000.00	0.00
South	40° 17'	41° 43'	99° 30'	39° 40'	2000000.00	0.00
New York						
Long Island	40° 40'	41° 02'	74° 00'	40° 30'	2000000.00	100000.00
North Carolina	34° 20'	36° 10'	79° 00'	33° 45'	2000000.00	0.00
North Dakota						
North	47° 26'	48° 44'	100° 30'	47° 00'	2000000.00	0.00
South	46° 11'	47° 29'	100° 30'	45° 40'	2000000.00	0.00
Ohio						
North	40° 26'	41° 42'	82° 30'	39° 40'	2000000.00	0.00
South	38° 44'	40° 02'	82° 30'	38° 00'	2000000.00	0.00
Oklahoma						
North	35° 34'	36° 46'	98° 00'	35° 00'	2000000.00	0.00
South	33° 56'	35° 14'	98° 00'	33° 20'	2000000.00	0.00
Oregon						
North	44° 20'	46° 00'	120° 30'	43° 40'	2000000.00	0.00
South	42° 20'	44° 00'	120° 30'	41° 40'	2000000.00	0.00
Pennsylvania						
North	40° 53'	41° 57'	77° 45'	40° 10'	2000000.00	0.00
South	39° 56'	40° 58'	77° 45'	39° 20'	2000000.00	0.00
Puerto Rico and Virgin Islands						
1	18° 02'	18° 26'	66° 26'	17° 50'	500000.00	0.00
2 (St. Croix)	18° 02'	18° 26'	66° 26'	17° 50'	500000.00	100000.00
Samoa	-14° 16'	-14° 16'	170° 00'		500000.00	0.00

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

	Standard Parallels		Origin Longitude	Origin Latitude	False Easting	False Northing
	ϕ_1 South	ϕ_2 North	λ_0 West	ϕ_0 North	E_0 (ft.)	N_0 (ft.)
South Carolina						
North	33° 46'	34° 58'	81° 00'	33° 00'	2000000.00	0.00
South	32° 20'	33° 40'	81° 00'	31° 50'	2000000.00	0.00
South Dakota						
North	44° 25'	45° 41'	100° 00'	43° 50'	2000000.00	0.00
South	42° 50'	44° 24'	100° 20'	42° 20'	2000000.00	0.00
Tennessee	35° 15'	36° 25'	86° 00'	34° 40'	2000000.00	100000.00
Texas						
North	34° 39'	36° 11'	101° 30'	34° 00'	2000000.00	0.00
North central	32° 08'	33° 58'	97° 30'	31° 40'	2000000.00	0.00
Central	30° 07'	31° 53'	100° 20'	29° 40'	2000000.00	0.00
South central	28° 23'	30° 17'	99° 00'	27° 50'	2000000.00	0.00
South	26° 10'	27° 50'	98° 30'	25° 40'	2000000.00	0.00
Utah						
North	40° 43'	41° 47'	111° 30'	40° 20'	2000000.00	0.00
Central	39° 01'	40° 39'	111° 30'	38° 20'	2000000.00	0.00
South	37° 13'	38° 21	111° 30'	36° 40'	2000000.00	0.00
Virginia						
North	38° 02'	39° 12'	78° 30'	37° 40'	2000000.00	0.00
South	36° 46'	37° 58'	78° 30'	36° 20'	2000000.00	0.00
Washington						
North	47° 30'	48° 44'	120° 50'	47° 00'	2000000.00	0.00
South	45° 50'	47° 20'	120° 30'	45° 20'	2000000.00	0.00
West Virginia						
North	39° 00'	40° 15'	79° 30'	38° 30'	2000000.00	0.00
South	37° 29'	38° 53'	81° 00'	37° 00'	2000000.00	0.00
Wisconsin						
North	45° 34'	46° 46'	90° 00'	45° 10'	2000000.00	0.00
Central	44° 15'	45° 30'	90° 00'	43° 50'	2000000.00	0.00
South	42° 44'	44° 04'	90° 00'	42° 00'	2000000.00	0.00

Lambert Conformal Conic Co-ordinates to Latitude/Longitude

Localization

If it is intended to do most conversions in the one SPCS zone, then the parameters for that zone can be coded into the program. When such a program is run, the program will prompt for the values (which allows the user to work in a different zone, as needed), but will display and store the regular values for the chosen zone. These can also be changed by changing the program, if a series of points on a different zone are to be converted.

The code required to 'hardwire' zone-specific values into the program is given below, based on a specific zone. If we were going to use California Zone III in SPCS 1983, its parameters are:

ϕ_0 = 36° 30' λ_0 = –120° 30' a = 6378137 m e^2 = 0.006 694 38

ϕ_1 = 37° 04' ϕ_2 = 38° 26' E_0 = 2,000,000.000 m N_0 = 500,000.000 m

The resulting code would be as follows, with the rest of the code left out. Note that the angular values are entered in HP notation (DDD.MMSS), as the program converts everything for internal use later.

Line	Instruction	Display	User Instructions
L001	LBL L		
………	……………		
L011	CL x		
L012	STO X		
L013	STO Y		
L014	36.3		ϕ_0 value of zone
L015	STO P		
L016	—120.30		λ_0 value of zone
L017	STO Q		
L018	37.04		ϕ_1 value of zone
L019	STO C		
L020	38.26		ϕ_2 value of zone
L021	STO D		
L022	2000000.0		E_0 value of zone
L023	STO G		
L024	500000.0		N_0 value of zone
L025	STO H		
L026	6378137		a value for ellipsoid (WGS84/NAD83)
L027	STO A		
L028	0.00669438		e^2 value for ellipsoid (WGS84/NAD83)
L029	STO E		
L030	CHECK–ENTER A		
L031	PSE		
L032	INPUT A		
	…………………		

This will change subsequent line numbers (they will be 6 greater than before), as well as the program length and checksum, but the program should otherwise be unaffected and should run correctly. Use the values for your preferred zone, and everything should be fine.

Corrections Line L079 changed to STO Z. Alaska standard parallel value corrected.