HP-35S Calculator Program — Utilities — 2A

Azimuth and Distance from Co-ordinates ('Inverse')

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Date: September, 2008. Version: 1.0 Mnemonic: **I** for '**I**nverse' Program

Line	Instruction	Display	User Instructions
I001	LBL I		LBL I
I002	CLSTK		CLEAR 5
I003	SF 10		FLAGS 1 .0
I004	COORD INVERSE		(Key in as EQN RCL C, RCL O, etc.; ENTER to end)
I005	PSE		PSE
I006	NTR FAR POINT		(Key in as EQN RCL N, RCL T, etc.; ENTER to end)
I007	PSE		PSE
I008	XEQ I029		
I009	RCL P		
I010	STO F		STO F
I011	NTR NEAR POINT		(Key in as EQN RCL N, RCL T, etc.; ENTER to end)
I012	PSE		PSE
I013	XEQ I029		
I014	RCL P		
I015	STO N		STO N
I016	RCL F		
I017	RCL– N		
I018	STO V		STO V
I019	ARG		ARG
I020	x ≥ 0?		x ? 0 5
I021	GTO I024		
I022	360		
I023	+		
I024	→HMS		→HMS
I025	RCL V		
I026	ABS		ABS
I027	STOP		Press R/S
I028	GTO I006		
I029	CLx		CLEAR 1
I030	STO N		STO N
I031	STO E		STO E
I032	KEY IN N		(Key in as EQN RCL K, RCL E, etc.; ENTER to end)
I033	PSE		PSE
I034	INPUT N		INPUT N
I035	KEY IN E		(Key in as EQN RCL K, RCL E, etc.; ENTER to end)
I036	PSE		PSE
I037	INPUT E		INPUT E
I038	RCL E		
I039	0 ***i*** 1		(Key in as 0, then **i**, then 1, press ENTER.)

Azimuth and Distance from Co-ordinates ('Inverse')

I040	×		
I041	RCL+ N		
I042	STO P		STO P
I043	RTN		RTN

This program is designed to hide completely the complex number work that the calculator performs to compute the azimuth and distance from the co-ordinates. All values entered and returned are presented in much the same manner as with the equivalent operation (Rectangular to Polar) in the HP-33S calculator.

Notes

(1) Set the calculator into DEGREES mode (press MODE 1) before starting, to make sure that you get degrees, minutes and seconds for the azimuth.

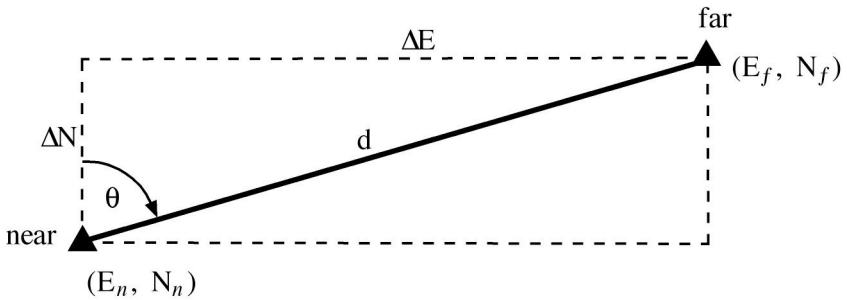
(2) This is a basic co-ordinate 'inverse' program that computes the azimuth and distance between two points whose co-ordinates are supplied. While the program works with complex number representations internally, this is hidden from the user.

(3) Azimuths are by themselves in HP notation, i.e., DDD.MMSSss.

(4) In order to display the prompts, this program sets Flag 10. However, the program never formally ends, because it is up to the user to decide when to stop and move control elsewhere. So the program never clears Flag 10. If you require Flag 10 to be clear, in order to process equations, you must do this manually.

Theory

If the co-ordinates of two points are given, there are two ways by which the azimuth and distance between them may be derived. These are equivalent, and will produce the same result. This HP-35s program uses the second method, viz., vectors stored as complex numbers.



1. **Using co-ordinates** alone, as in the figure above the differences between the easting co-ordinates of the points (ΔE) and difference between the northing co-ordinates of the two points (ΔN) are obtained. The point from which the azimuth is desired to come is termed the 'near' point, while the point to which the azimuth points is termed the 'far' point. So:

$$\Delta E = E_f - E_n$$

$$\Delta N = N_f - N_n$$

Azimuth and Distance from Co-ordinates ('Inverse')

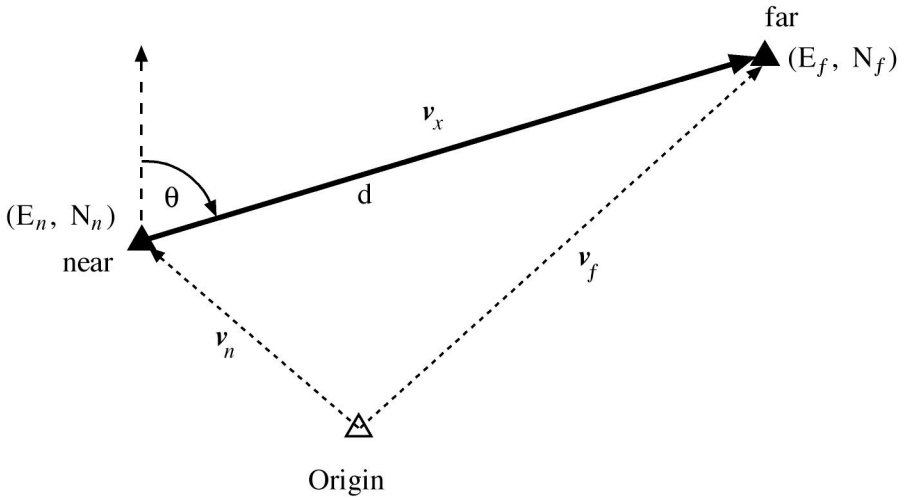
The distance, d, between the two points is computed using:

$$d = \sqrt{\Delta E^2 + \Delta N^2}$$

The azimuth, θ, between the two points is computed using:

$$\theta = \arctan\left(\frac{\Delta E}{\Delta N}\right)$$

The ATAN2 function (in some programming languages and Excel) takes the two components, ΔE and ΔN, as separate arguments, and returns an azimuth in the range –180° to +180°. This is easily converted to an azimuth in the range 0° to 360° The HP calculator function R→P performed a similar process, but it is not present on the HP-35s, so the angle must be put into its correct quadrant manually, as the ATAN function returns a value between –90° and +90°.



2. **Using vectors**, as shown in the figure above, the co-ordinates of each point are entered as the components of a pair of 2-D vectors. In the HP-35s, 2-D vectors are best handled as complex numbers, so that the co-ordinates are stored as the two vectors from the origin, $\mathbf{v}_f$ and $\mathbf{v}_n$, as follows:

$$\mathbf{v}_f = N_f + i\ E_f$$

$$\mathbf{v}_n = N_n + i\ E_n$$

The vector from the near point to the far point, $\mathbf{v}_x$, is then the difference between the two vectors:

$$\mathbf{v}_x = \mathbf{v}_f - \mathbf{v}_n$$

This calculation is handled by a simple vector (albeit using complex numbers) subtraction in the HP-35s.

To obtain the length of the vector $\mathbf{v}_x$, its absolute value must be taken (using the ABS function):

$$d = |\mathbf{v}_x|$$

Azimuth and Distance from Co-ordinates ('Inverse')

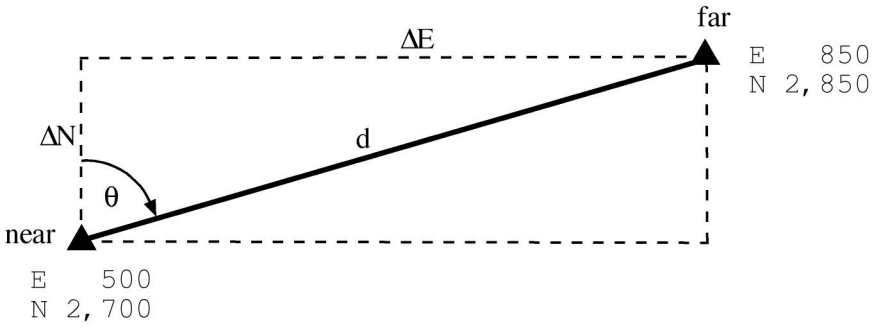
The azimuth of the vector $\mathbf{v}_x$ is calculated as the argument of the vector, using the ARG function:

$$\theta = \arg(\mathbf{v}_x)$$

The HP-35s returns the azimuth in the units in which the calculator us currently set, usually degrees. The value will be in the range –180° to +180°. The program brings this value into the range 0° to 360°, and converts it to degrees, minutes and seconds, in HP notation (i.e., HHH.MMSSsss).

Sample Computations and Running the Program

Example 1



In the above example, $\Delta E = 350$ and $\Delta N = 150$. Using the first method discussed above:

$$d = \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{122{,}500 + 22{,}500} = \sqrt{145{,}000} = 380.789$$

$$\theta = \arctan\left(\frac{\Delta E}{\Delta N}\right) = \arctan\left(\frac{350}{150}\right) = \arctan(2.333\ldots) = 66° 48' 05"$$

Using the second method, the complex numbers formed are:

$$\mathbf{v}_f = 2{,}850 + \boldsymbol{i}\ 850$$
$$\mathbf{v}_n = 2{,}700 + \boldsymbol{i}\ 500$$

and the difference between them is:

$$\mathbf{v}_x = 150 + \boldsymbol{i}\ 350$$

so $d = |\mathbf{v}_x| = \sqrt{350^2 + 150^2} = \sqrt{122{,}500 + 22{,}500} = \sqrt{145{,}000} = 380.789$

and $\theta = \arg(\mathbf{v}_x) = 66° 48' 05"$

Using the calculator for this process, begin by pressing XEQ I and then the Enter key.

The calculator briefly displays COORD INVERSE, to indicate the program, then NTR FAR POINT, to tell the user to enter the co-ordinates of the far point, and finally KEY IN N. The calculator then prompts for entry of the northing co-ordinate of the far point, as follows:

N?

0.0000

Key in the northing of the far point, in this case 2850, and press R/S.

The calculator briefly displays KEY IN E, then prompts for keying in the easting of the far point:

E?
0.0000

Key in the easting of the far point, in this case 850, and press R/S.

The calculator briefly displays NTR NEAR POINT, then KEY IN N, then prompts the user to key in the northing of the near point, displaying:

N?
0.0000

Key in the northing of the near point, in this case 2700, and press R/S.

The calculator briefly displays KEY IN E, then prompts for keying in the easting of the near point:

E?
0.0000

Key in the easting of the near point, in this case 500, and press R/S.

The calculator then displays the azimuth of the line in line 1 (the upper line of the display) in HP notation (DDD.MMSSsss), and the length of the line in line 2 (the lower line of the display. In this case, the display shows:

66.4805	(meaning 66° 48' 05", the azimuth of the line)
380.7887	(the length of the line)

At this point, if the user presses R/S, the program prompts for the northing of the far point, as discussed above. Otherwise, the user can keep working on whatever they were doing before. Note that Flag 10 will now be set.

Example 2

Near Point	E 500		Far Point	E 850
	N 2,700			N 2,600

Begin by pressing XEQ I and then the Enter key.

The calculator briefly displays COORD INVERSE, to indicate the program, then NTR FAR POINT, to tell the user to enter the co-ordinates of the far point, and finally KEY IN N. The calculator then prompts for entry of the northing co-ordinate of the far point, as follows:

N?
0.0000

Key in the northing of the far point, in this case 2600, and press R/S.

The calculator briefly displays KEY IN E, then prompts for keying in the easting of the far point:

E?
0.0000

Key in the easting of the far point, in this case 850, and press R/S.

The calculator briefly displays NTR NEAR POINT, then KEY IN N, then prompts the user to key in the northing of the near point, displaying:

N?
0.0000

Key in the northing of the near point, in this case 2700, and press R/S.

The calculator briefly displays KEY IN E, then prompts for keying in the easting of the near point:

E?
0.0000

Key in the easting of the near point, in this case 500, and press R/S.

The calculator then displays the azimuth of the line in line 1 (the upper line of the display) in HP notation (DDD.MMSSsss), and the length of the line in line 2 (the lower line of the display. In this case, the display shows:

105.5643	(meaning 105° 56' 43", the azimuth of the line)
364.0055	(the length of the line)

At this point, if the user presses R/S, the program prompts for the northing of the far point, as discussed above. Otherwise, the user can keep working on whatever they were doing before. Note that Flag 10 will now be set.

Distance = 364.0055 Azimuth = 105° 56' 43"

Example 3

Near Point	E 500	Far Point	E 450
	N 2,700		N 2,500

Process the co-ordinates through the calculator as above, to get the following results:

Distance = 206.1553 Azimuth = 194° 02' 10"

Azimuth and Distance from Co-ordinates ('Inverse')

Example 4

Near Point	E 500	Far Point	E 300
	N 2,700		N 2,800

Process the co-ordinates through the calculator as above, to get the following results:

Distance = 223.6068 Azimuth = 296° 33' 54"

Storage Registers Used

E Easting co-ordinates of the entered points.

F Co-ordinates of far point, as a complex number.

N Northing co-ordinate of the entered points, then the co-ordinates of the near point, as a complex number.

P Temporary storage of co-ordinates of entered point, as a complex number.

V Vector from the near point to the far point, stored as a complex number.

Statistical Registers: not used.

Labels Used

Label **I** Length = 191 Checksum = BA0E

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Routines Called

None. The program does call the same segment of code twice, for co-ordinate entry, but this is within the program.

Flags Used

The program sets Flag 10, to allow the display of prompts. However, as the program never formally ends (it is used by the user as needed), it never clears this flag. If the flag is needed to be cleared, clear it manually.