Plane Triangle Solutions

1. Given 3 sides, compute the three angles and area

Programmer: Dr. Bill Hazelton

Line	Instruction	Display	Program Entry Instructions
A0001	LBL A		
A0002	SF 10		FLAGS SF .0
A0003	ENTER SIDE 1		EQN RCL E RCL N etc. ENTER to end
A0004	PSE		
A0005	INPUT S		
A0006	STO A		
A0007	ENTER SIDE 2		EQN RCL E RCL N etc. ENTER to end
A0008	PSE		
A0009	INPUT S		
A0010	STO B		
A0011	ENTER SIDE 3		EQN RCL E RCL N etc. ENTER to end
A0012	PSE		
A0013	INPUT S		
A0014	STO C		
A0015	\mathbf{x}^2		
A0016	RCL B		
A0017	\mathbf{x}^2		
A0018	+		
A0019	RCL A		
A0020	x ²		
A0021	-		
A0022	RCL÷ B		
A0023	RCL÷ C		
A0024	2		
A0025	÷		
A0026	ACOS		
A0027	→HMS		<u></u>
A0028	ANGLE 1 =		EQN RCL A RCL N etc. ENTER to end
A0029	PSE		
A0030	STOP		
A0031	RCL C		
A0032	x ²		
A0033	RCL A		
A0034	x ²		
A0035	+		
A0036	RCL B		
A0037	x ²		

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A0038	_	
A0039	RCL÷ C	
A0040	RCL÷ A	
A0041	2	
A0042	÷	
A0043	ACOS	
A0044	→HMS	
A0045	ANGLE 2 =	EQN RCL A RCL N etc. ENTER to end
A0046	PSE	
A0047	STOP	
A0048	RCL B	
A0049	x ²	
A0050	RCL A	
A0051	x ²	
A0052	+	
A0053	RCL C	
A0054	x ²	
A0055	-	
A0056	RCL÷ A	
A0057	RCL÷ B	
A0058	2	
A0059	÷	
A0060	ACOS	
A0061	STO D	
A0062	→HMS	_
A0063	ANGLE 3 =	EQN RCL A RCL N etc. ENTER to end
A0064	PSE	
A0065	STOP	
A0066	RCL D	
A0067	SIN	
A0068	2	
A0069	÷	
A0070	RCLx A	
A0071	RCLx B	
A0072	AREA =	EQN RCL A RCL R etc. ENTER to end
A0073	PSE	_
A0074	STOP	
A0075	CF 10	FLAGS CF .0
A0076	RTN	

Label Used

Label A Length = 345 Checksum = 3DD1

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

Plane Triangle Solutions

Triangles 1

Storage Registers Used

- A Side length 1
- **B** Side length 2
- C Side length 3
- **D** Angle 3 in decimal degrees
- **S** Temporary storage of input side length

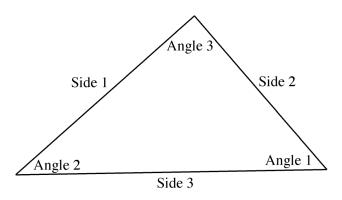
Notes

- (1) Program for computing the three angles and area of a plane triangle, given the lengths of the three sides.
- (2) Angles are displayed in HP notation, i.e., DDD.MMSS.
- (3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
- (4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.

Theory

This program accepts the three side lengths of the triangle, then uses to cosine rule to compute the three angles. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number.



The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to 180°.

Running the Program

Press XEQ A

Screen shows ENTER SIDE 1 briefly, then prompts with S?

Enter the length of side 1 and press R/S.

Screen shows ENTER SIDE 2 briefly, then prompts with S?

Enter the length of side 2 and press R/S.

Screen shows ENTER SIDE 3 briefly, then prompts with S?

Enter the length of side 3 and press R/S.

Screen shows ANGLE 1 =briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This clears Flag 10, which was set at the start of the program.

Sample Computations

	Triangle 1	Triangle 2
Inputs:	Side Length $1 = 100.000$	Side Length $1 = 10.000$
	Side Length $2 = 100.000$	Side Length $2 = 10.000$
	Side Length $3 = 100.000$	Side Length $3 = 18.000$
Results :	Angle 1 = $60^{\circ} 00' 00''$	Angle 1 = $25^{\circ} 50' 31''$
	Angle 2 = $60^{\circ} \ 00' \ 00''$	Angle 2 = $25^{\circ} 50' 31''$
	Angle $3 = 60^{\circ} 00' 00''$	Angle $3 = 128^{\circ} 18' 58''$
	Area = 4,330.127	Area = 39.230
	Check angle sum = $180^{\circ} 00' 00"$	Check = $180^{\circ} 00' 00''$

Plane Triangle Solutions

2. Given 2 sides and included angle, compute the other two angles, other side and the area

Programmer: Dr. Bill Hazelton

Line	Instruction	Display	Program Entry Instructions
B0001	LBL B	*	
B0002	SF 10		➡ FLAGS SF .0
B0003	ENTER SIDE 1		EQN RCL E RCL N etc. ENTER to end
B0004	PSE		
B0005	INPUT S		
B0006	STO A		
B0007	ENTER SIDE 2		EQN RCL E RCL N etc. ENTER to end
B0008	PSE		
B0009	INPUT S		
B0010	STO B		
B0011	ENTER ANGLE 3		EQN RCL E RCL N etc. ENTER to end
B0012	PSE		
B0013	INPUT S		
B0014	→HR		
B0015	STO C		
B0016	COS		
B0017	RCLx A		
B0018	RCLx B		
B0019	2		
B0020	Х		
B0021	RCL A		
B0022	\mathbf{x}^2		
B0023	x <> y		
B0024	_		
B0025	RCL B		
B0026	x ²		
B0027	+		
B0028	\sqrt{X}		
B0029	STO D		
B0030	SIDE 3 =		EQN RCL S RCL I etc. ENTER to end
B0031	PSE		
B0032	STOP		
B0033	RCL C		
B0034	SIN		
B0035	RCL÷ D		
B0036	STO D		
B0037	RCLx A		
B0038	ASIN		
B0039	→HMS		

B0040ANGLE 1 =Image: Eqn RCL A RCL N etc. ENTER to endB0041PSEImage: STOPB0042STOPImage: STOPB0043RCL DImage: STOPB0044RCLx BImage: STOPB0045ASINImage: STOPB0046 \rightarrow HMSImage: STOPB0047ANGLE 2 =Image: STOPB0048PSEImage: STOPB0050RCL CImage: STOPB0051SINImage: SINB0052RCLx AImage: STOPB0053RCLx BImage: STOPB00542Image: STOPB0055 \div Image: STOPB0056AREA =Image: STOPB0058STOPImage: STOPB0059CF 10Image: STOPB0050RTNImage: STOP			
B0042 STOP B0043 RCL D B0044 RCLx B B0045 ASIN B0046 \rightarrow HMS B0047 ANGLE 2 = B0048 PSE B0049 STOP B0050 RCL C B0051 SIN B0052 RCLx A B0053 RCLx B B0054 2 B0055 \div B0056 AREA = B0058 STOP B0058 STOP B0059 CF 10	B0040	ANGLE 1 =	EQN RCL A RCL N etc. ENTER to end
B0043RCL DB0044RCLx BB0045ASINB0046 \rightarrow HMSB0047ANGLE 2 =B0048PSEB0049STOPB0050RCL CB0051SINB0052RCLx AB0053RCLx BB0056AREA =B0057PSEB0058STOPB0059CF 10CF 10PLAGS CF .0	B0041	PSE	
B0044RCLx BB0045ASINB0046→HMSB0047ANGLE 2 =B0048PSEB0049STOPB0050RCL CB0051SINB0052RCLx AB0053RCLx BB0055÷B0056AREA =B0057PSEB0058STOPB0059CF 10CF 10FLAGS CF .0	B0042	STOP	
B0045ASINB0046 \rightarrow HMSB0047ANGLE 2 =B0047ANGLE 2 =B0048PSEB0049STOPB0050RCL CB0051SINB0052RCLx AB0053RCLx BB00542B0055 \div B0056AREA =B0057PSEB0058STOPB0059CF 10CF 10PLAGS CF .0	B0043	RCL D	
B0046 \rightarrow HMSB0047ANGLE 2 =B0048PSEB0049STOPB0050RCL CB0051SINB0052RCLx AB0053RCLx BB0055 \div B0056AREA =B0057PSEB0058STOPB0059CF 10CF 10 \checkmark FLAGS CF .0	B0044	RCLx B	
B0047ANGLE 2 =B0048PSEB0049STOPB0050RCL CB0051SINB0052RCLx AB0053RCLx BB00542B0055 \div B0056AREA =B0057PSEB0058STOPB0059CF 10CF 10 \checkmark FLAGS CF .0	B0045	ASIN	
B0048PSEB0049STOPB0050RCL CB0051SINB0052RCLx AB0053RCLx BB00542B0055 \div B0056AREA =B0057PSEB0058STOPB0059CF 10CF 10FLAGS CF .0	B0046	→HMS	
B0049 STOP B0050 RCL C B0051 SIN B0052 RCLx A B0053 RCLx B B0054 2 B0055 ÷ B0056 AREA = B0057 PSE B0058 STOP B0059 CF 10	B0047	ANGLE 2 =	EQN RCL A RCL N etc. ENTER to end
B0050 RCL C B0051 SIN B0052 RCLx A B0053 RCLx B B0054 2 B0055 ÷ B0056 AREA = B0057 PSE B0058 STOP B0059 CF 10	B0048	PSE	
B0051SINB0052RCLx AB0053RCLx BB00542B0055 \div B0056AREA =B0057PSEB0058STOPB0059CF 10CF 10FLAGS CF .0	B0049	STOP	
B0052 RCLx A B0053 RCLx B B0054 2 B0055 ÷ B0056 AREA = B0057 PSE B0058 STOP B0059 CF 10	B0050	RCL C	
B0053 RCLx B B0054 2 B0055 ÷ B0056 AREA = B0057 PSE B0058 STOP B0059 CF 10 FLAGS CF .0	B0051	SIN	
B0054 2 B0055 ÷ B0056 AREA = B0057 PSE B0058 STOP B0059 CF 10	B0052	RCLx A	
B0055 ÷ B0056 AREA = B0057 PSE B0058 STOP B0059 CF 10 FLAGS CF .0	B0053	RCLx B	
B0056 AREA = E EQN RCL A RCL R etc. ENTER to end B0057 PSE E E E B0058 STOP E E FLAGS CF .0	B0054	2	
B0057 PSE B0058 STOP B0059 CF 10	B0055	÷	
B0058 STOP B0059 CF 10	B0056	AREA =	EQN RCL A RCL R etc. ENTER to end
B0059 CF 10 FLAGS CF .0	B0057	PSE	
	B0058	STOP	<u> </u>
B0060 RTN	B0059	CF 10	FLAGS CF .0
	B0060	RTN	

Plane Triangle Solutions

Notes

- (1) Program for computing two angles, one side and the area of a plane triangle, given the lengths of two sides and the angle between them.
- (2) Angles are displayed in HP notation, i.e., DDD.MMSS, and should also be entered in that format.
- (3) Whatever linear units are used (and they should be the same for all the sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
- (4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.

Storage Registers Used

- A Side length 1
- **B** Side length 2
- **C** Angle 3 in decimal degrees
- **D** Side 3 temporary storage, then sine ratio temporary storage
- **S** Temporary input storage

Label Used

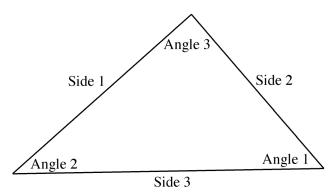
Label **B** Length = 273 Checksum = 927C

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

Theory

This program accepts the two known side lengths of the triangle, followed by the included angle, then uses to cosine rule to compute the other side. The sine rule is used to compute the other two angles. The area is computed using the length of the two given sides and half the sine of the given angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. Side 1 is assumed to be to the right of the included known angle, here angle 3. Side 2 is to the left of the included angle.



The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to 180°.

Sample Computations			
	Triangle 1	Triangle 2	
Inputs:	Side Length $1 = 100.000$	Side Length $1 = 10.000$	
	Side Length $2 = 100.000$	Side Length $2 = 10.000$	
	Angle $3 = 60^{\circ} 00' 00''$	Angle 3 = 128° 18' 58"	
Results :	Side $3 = 100.000$	Side $3 = 18.000$	
	Angle 1 = $60^{\circ} \ 00' \ 00''$	Angle 1 = $25^{\circ} 50' 31''$	
	Angle 2 = $60^{\circ} \ 00' \ 00''$	Angle 2 = $25^{\circ} 50' 31''$	
	Area = 4,330.127	Area = 39.230	
	Check angle sum = $180^{\circ} 00' 00"$	Check = $180^{\circ} 00' 00''$	

Triangles 1

Running the Program

Press XEQ B

Screen shows ENTER SIDE 1 briefly, then prompts with S?

Enter the length of side 1 and press R/S.

Screen shows ENTER SIDE 2 briefly, then prompts with S?

Enter the length of side 2 and press R/S.

Screen shows ENTER ANGLE 3 briefly, then prompts with S?

Enter angle 3 in HP notation and press R/S.

Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 1 =briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This clears Flag 10, which was set at the start of the program.

Plane Triangle Solutions

3. Given 2 angles and the included side, compute the other angle, the other 2 sides and area

Programmer: Dr. Bill Hazelton

Line	Instruction	Display	Program Entry Instructions
C0001	LBL C		
C0002	SF 10		FLAGS SF .0
C0003	ENTER ANGLE 1		EQN RCL E RCL N etc. ENTER to end
C0004	PSE		
C0005	INPUT S		
C0006	→HR		
C0007	STO A		
C0008	ENTER ANGLE 2		EQN RCL E RCL N etc. ENTER to end
C0009	PSE		
C0010	INPUT S		
C0011	→HR		
C0012	STO B		
C0013	ENTER SIDE 3		EQN RCL E RCL N etc. ENTER to end
C0014	PSE		
C0015	INPUT S		
C0016	STO C		
C0017	180		
C0018	RCL- A		
C0019	RCL- B		
C0020	STO D		
C0021	→HMS		
C0022	ANGLE 3 =		EQN RCL A RCL N etc. ENTER to end
C0023	PSE		
C0024	STOP		
C0025	RCL C		
C0026	RCL D		
C0027	SIN		
C0028	STO E		
C0029	÷		
C0030	STO F		
C0031	RCL A		
C0032	SIN		
C0033	X		
C0034	STOx E		
C0035	SIDE 1 =		EQN RCL S RCL I etc. ENTER to end
C0036	PSE		
C0037	STOP		
C0038	RCL F		
C0039	RCL B]

C0040	SIN	
C0041	X	
C0042	STOx E	
C0043	SIDE 2 =	EQN RCL S RCL I etc. ENTER to end
C0044	PSE	
C0045	STOP	
C0046	RCL E	
C0047	2	
C0048	÷	
C0049	AREA =	EQN RCL A RCL R etc. ENTER to end
C0050	PSE	
C0051	STOP	
C0052	CF 10	FLAGS CF .0
C0053	RTN	

Notes

- (1) Program for computing two side, an angle and area of a plane triangle, given two angles and the length of the side between them.
- (2) Angles are displayed in HP notation, i.e., DDD.MMSS. They should also be entered in this format.
- (3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
- (4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.
- (5) This program is essentially a surveying 'intersection' problem, and can also be interpreted as the 'two missing sides' problem.

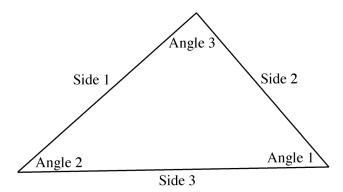
Storage Registers Used

- A Angle 1 in decimal degrees
- **B** Angle 2 in decimal degrees
- C Length of Side 3
- **D** Angle 3 temporary storage, in decimal degrees
- **E** Area accumulation variable
- **F** Sine ratio temporary storage
- **S** Temporary input storage

Theory

This program accepts the two known angles of the triangle, then the length of the side between them. It then computes the remaining angle by subtracting the sum of the two known angles from 180°. The sine rule is used to compute the other two sides. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. It is assumes that Angle 1 and Angle 2 are known and entered in that order, and side 3 is the side between them.



The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to 180°.

Sample Computations

	Triangle 1	Triangle 2
Inputs:	Angle 1 = $60^{\circ} 00' 00''$	Angle 1 = $25^{\circ} 50' 31''$
	Angle 2 = $60^{\circ} 00' 00"$	Angle 2 = $25^{\circ} 50' 31''$
	Side Length $3 = 100.000$	Side Length $3 = 18.000$
Results :	Angle $3 = 60^{\circ} 00' 00''$	Angle $3 = 128^{\circ} 18' 58''$
	Side $1 = 100.000$	Side $1 = 10.000$
	Side $2 = 100.000$	Side $2 = 10.000$
	Area = 4,330.127	Area = 39.230
	Check angle sum = $180^{\circ} 00' 00"$	Check = $180^{\circ} 00' 00''$

Label Used

Label C	Length = 252	Checksum = $A47F$
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Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

Running the Program

Press XEQ C

Screen shows ENTER ANGLE 1 briefly, then prompts with S?

Enter angle 1 in HP notation and press R/S.

Screen shows ENTER ANGLE 2 briefly, then prompts with S?

Enter angle 2 in HP notation and press R/S.

Screen shows ENTER SIDE 3 briefly, then prompts with S?

Enter the length of side 3 and press R/S.

Screen shows ANGLE 3 = briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 2 = briefly, then shows Side 2 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This clears Flag 10, which was set at the start of the program.

Plane Triangle Solutions

4. Given 2 angles and a non-included side, compute the other angle, the other two sides and area

Programmer: Dr. Bill Hazelton

Line	Instruction	Display	Program Entry Instructions
D0001	LBL D	. .	
D0002	SF 10		FLAGS SF .0
D0003	ENTER ANGLE 1		EQN RCL E RCL N etc. ENTER to end
D0004	PSE		
D0005	INPUT S		
D0006	→HR		
D0007	STO A		
D0008	ENTER ANGLE 3		EQN RCL E RCL N etc. ENTER to end
D0009	PSE		
D0010	INPUT S		
D0011	→HR		
D0012	STO B		
D0013	ENTER SIDE 3		EQN RCL E RCL N etc. ENTER to end
D0014	PSE		
D0015	INPUT S		
D0016	STO C		
D0017	RCL B		
D0018	SIN		
D0019	÷		
D0020	STO D		
D0021	RCL A		
D0022	SIN		
D0023	Х		
D0024	SIDE 1 =		EQN RCL S RCL I etc. ENTER to end
D0025	PSE		
D0026	STOP		
D0027	180		
D0028	RCL- A		
D0029	RCL- B		
D0030	STO E		
D0031	→HMS		
D0032	ANGLE 2 =		EQN RCL A RCL N etc. ENTER to end
D0033	PSE		
D0034	STOP		
D0035	RCL E		
D0036	SIN		
D0037	RCLx D		
D0038	STO E]
D0039	SIDE 2 =		EQN RCL S RCL I etc. ENTER to end

D0040	PSE	
D0041	STOP	
D0042	RCL E	
D0043	RCLx C	
D0044	RCL A	
D0045	SIN	
D0046	X	
D0047	2	
D0048	÷	
D0049	AREA =	EQN RCL A RCL R etc. ENTER to end
D0050	PSE	
D0051	STOP	
D0052	CF 10	FLAGS CF .0
D0053	RTN	

Notes

- (1) Program for computing two side, an angle and area of a plane triangle, given two angles and the length of a side not between them.
- (2) Angles are displayed in HP notation, i.e., DDD.MMSS. They should also be entered in this format.
- (3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
- (4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.

Storage Registers Used

- A Angle 1 in decimal degrees
- **B** Angle 3 in decimal degrees
- C Length of Side 3
- **D** Sine ratio temporary storage
- E Angle 2 temporary storage, in decimal degrees, then side 2 temporary storage
- **S** Temporary input storage

Label Used

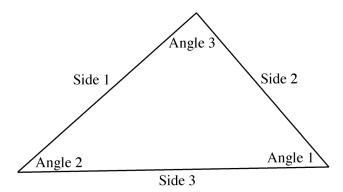
Label **D** Length = 252 Checksum = 5939

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

Theory

This program accepts the two known angles of the triangle, then the length of a side not between them. It then computes the remaining angle by subtracting the sum of the two known angles from 180°. The sine rule is used to compute the other two sides. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. It is assumes that Angle 1 and Angle 3 are known and entered in that order, and side 3 is the known side.



The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to 180°.

Sample Computations

Triangle 1 Triangle 2 Inputs: Angle 1 = $60^{\circ} 00' 00''$ Angle $1 = 25^{\circ} 50' 31''$ Angle $3 = 60^{\circ} 00' 00''$ Angle $3 = 128^{\circ} 18' 58''$ Side Length 3 = 100.000Side Length 3 = 18.000**Results**: Side 1 = 100.000Side 1 = 10.000Angle $2 = 60^{\circ} 00' 00''$ Angle $2 = 25^{\circ} 50' 31''$ Side 2 = 100.000Side 2 = 10.000Area = 4,330.127Area = 39.230Check angle sum = $180^{\circ} 00' 00''$ Check = $180^{\circ} 00' 00''$

Running the Program

Press XEQ D

Screen shows ENTER ANGLE 1 briefly, then prompts with S?

Enter angle 1 in HP notation and press R/S.

Screen shows ENTER ANGLE 3 briefly, then prompts with S?

Enter angle 3 in HP notation and press R/S.

Screen shows ENTER SIDE 3 briefly, then prompts with S?

Enter the length of side 3 and press R/S.

Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 1 =briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 2 = briefly, then shows Side 2 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This clears Flag 10, which was set at the start of the program.

Plane Triangle Solutions

5. Given 2 sides and an angle not between them, compute the other two angles, the other side and area (two possible solutions)

Programmer: Dr. Bill Hazelton

Line	Instruction	Display	Program Entry Instructions
E0001	LBL E		
E0002	SF 10		FLAGS SF .0
E0003	ENTER SIDE 1		EQN RCL E RCL N etc. ENTER to end
E0004	PSE		
E0005	INPUT S		
E0006	STO A		
E0007	ENTER SIDE 2		EQN RCL E RCL N etc. ENTER to end
E0008	PSE		
E0009	INPUT S		
E0010	STO B		
E0011	ENTER ANGLE 1		EQN RCL E RCL N etc. ENTER to end
E0012	PSE		
E0013	INPUT S		
E0014	→HR		
E0015	STO C		
E0016	SIN		
E0017	RCL÷ A		
E0018	STO D		
E0019	RCLx B		
E0020	ASIN		
E0021	STO E		Angle 2, solution 1
E0022	180		
E0023	x <> y		
E0024	_		
E0025	RCL- C		
E0026	STO F		Angle 3, solution 1
E0027	SIN		
E0028	RCLx A		
E0029	RCL C		
E0030	SIN		
E0031	÷		
E0032	STO G		Side 3, solution 1
E0033	RCL F		
E0034	SIN		
E0035	RCLx A		
E0036	RCLx B		
E0037	2		
E0038	÷		

Plane Triangle Solutions

	~~~~	
E0039	STO H	Area, solution 1
E0040	SOLUTION 1	EQN RCL S RCL O etc. ENTER to end
E0041	PSE	
E0042	ANGLE 2 =	EQN RCL A RCL N etc. ENTER to end
E0043	PSE	
E0044	RCL E	
E0045	→HMS	
E0046	STOP	
E0047	ANGLE 3 =	EQN RCL A RCL N etc. ENTER to end
E0048	PSE	
E0049	RCL F	
E0050	→HMS	
E0051	STOP	
E0052	SIDE 3 =	EQN RCL S RCL I etc. ENTER to end
E0053	PSE	
E0054	RCL G	
E0055	STOP	
E0056	AREA =	EQN RCL A RCL R etc. ENTER to end
E0057	PSE	
E0058	RCL H	
E0059	STOP	
E0060	180	
E0061	RCL- E	
E0062	STO E	Angle 2, solution 2
E0063	180	Angle 2, solution 2
E0064		
E0065	x <> y	
E0065	RCL- C	
E0067	STO F	Angle 2 solution 2
E0067 E0068	SIN	Angle 3, solution 2
E0069	RCLx A	
E0070	RCL C	
E0071	SIN	
E0072	÷	Side 2 colution 2
E0073	STO G	Side 3, solution 2
E0074	RCL F	
E0075	SIN	
E0076	RCLx A	
E0077	RCLx B	
E0078	2	
E0079	÷	
E0080	STO H	Area, solution 2
E0081	SOLUTION 2	EQN RCL S RCL O etc. ENTER to end
E0082	PSE	
E0083	ANGLE 2 =	EQN RCL A RCL N etc. ENTER to end
E0084	PSE	
E0085	RCL E	

Fiane	Thangle 3	
E0086	→HMS	
E0087	STOP	
E0088	ANGLE 3 =	EQN RCL A RCL N etc. ENTER to end
E0089	PSE	
E0090	RCL F	
E0091	→HMS	
E0092	STOP	
E0093	SIDE 3 =	EQN RCL S RCL I etc. ENTER to end
E0094	PSE	
E0095	RCL G	
E0096	STOP	
E0097	AREA =	EQN RCL A RCL R etc. ENTER to end
E0098	PSE	
E0099	RCL H	
E0100	STOP	
E0101	CF 10	FLAGS CF .0
E0102	RTN	

### Label Used

Label E Length = $487$ Checksum = $E72F$
------------------------------------------

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

### Notes

- (1) Program for computing the two angles, a side and area of a plane triangle, given the lengths of two sides and an angle not between the two sides.
- (2) Angles are displayed in HP notation, i.e., DDD.MMSS.
- (3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
- (4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.
- (5) There are two possible solutions to this triangle. Each solution is presented separately. Note that if one of the solutions is not physically possible (usually solution 2), the program may return an error.

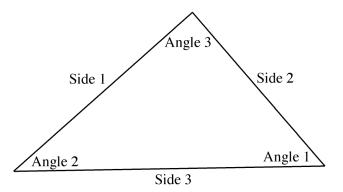
### **Storage Registers Used**

Α	Side length 1
В	Side length 2
С	Angle 3
D	Temporary storage for sine ratio
Ε	Angle 2 results
F	Angle 3 results
G	Side 3 results
Η	Area results
S	Temporary storage of input side length

### Theory

This program accepts two side lengths of the triangle and an angle not between them, then uses the sine rule to compute one of the other angles. The third angle is computed by subtracting the sum of the other angles from 180°. The remaining side is computed using the sine rule. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. In this case, Side 1 and Side 2 are known, along with Angle 1.



There are two possible solutions, depending upon the solutions to Angle 2. Because it is determined using the sine rule, and  $\arcsin(x)$  can have multiple values, there is one solution where Angle 2 lies between 0° and 90°, and a second where Angle 2 lies between 90° and 180°. Both these solutions are computed. The results are presented in two groups, with suitable text prompts.

Solution 1 is based on Angle 2 being less than 90°. Solution 2 is based on Angle 2 being greater than 90°.

The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to 180°.

### **Running the Program**

Press XEQ E

Screen shows ENTER SIDE 1 briefly, then prompts with S?

Enter the length of side 1 and press R/S.

Screen shows ENTER SIDE 2 briefly, then prompts with S?

Enter the length of side 2 and press R/S.

Screen shows ENTER ANGLE 1 briefly, then prompts with S?

Enter angle 1 in HP notation and press R/S.

Screen shows SOLUTION 1 briefly.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 3 = briefly, then shows Side 3 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to continue.

Screen shows SOLUTION 2 briefly.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 3 = briefly, then shows Side 3 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This clears Flag 10, which was set at the start of the program.

### Triangles 1

### **Sample Computations**

	Triangle 1	Triangle 2
Inputs:	Side Length $1 = 100.000$	Side Length $1 = 10.000$
	Side Length $2 = 100.000$	Side Length $2 = 10.000$
	Angle 1 = $60^{\circ} 00' 00''$	Angle 1 = $25^{\circ} 50' 31''$
<b>Results</b> :	Solution 1	
	Angle 2 = $60^{\circ} 00' 00''$	Angle 2 = $25^{\circ} 50' 31''$
	Angle $3 = 60^{\circ} 00' 00''$	Angle $3 = 128^{\circ} 18' 58''$
	Side Length $3 = 100.000$	Side Length $3 = 18.000$
	Area = 4,330.127	Area = 39.230
	[Check angle sum = $180^{\circ} 00' 00"$	Check = $180^{\circ} 00' 00"$ ]
	Solution 2	
	Angle $2 = 120^{\circ} 00' 00"$	Angle $2 = 154^{\circ} 09' 29''$
	Angle $3 = 0^{\circ} 00' 00''$	Angle $3 = 0^{\circ} 00' 00''$ (very small)
	Side Length $3 = 0.000$	Side Length $3 = 0.000$
	Area = $0.000$	Area = $0.000$
	Check angle sum = $180^{\circ} 00' 00"$	Check = $180^{\circ} 00' 00''$

Clearly, neither of the results for Solution 2 are particularly good solutions (despite being mathematically valid and correct), because of the zero angles involved. This, as well as negative angles, is one way to spot an unsuitable solution. However, it is possible to have two perfectly reasonable solutions, in which case you will need to look beyond the given data to decide which is the required solution.