## 1. Given 3 sides, compute the three angles and area

Programmer: Dr. Bill Hazelton
Date: December, 2005.

| Line | Instruction | Display | Program Entry Instructions |
| :---: | :---: | :---: | :---: |
| A0001 | LBL A |  |  |
| A0002 | SF 10 |  | $\square$ FLAGS SF . 0 |
| A0003 | ENTER SIDE 1 |  | $\rightarrow$ EQN RCL E RCL N etc. ENTER to end |
| A0004 | PSE |  |  |
| A0005 | INPUT S |  |  |
| A0006 | STO A |  |  |
| A0007 | ENTER SIDE 2 |  | $\mapsto$ EQN RCL E RCL N etc. ENTER to end |
| A0008 | PSE |  |  |
| A0009 | INPUT S |  |  |
| A0010 | STO B |  |  |
| A0011 | ENTER SIDE 3 |  | $\longrightarrow$ EQN RCL E RCL N etc. ENTER to end |
| A0012 | PSE |  |  |
| A0013 | INPUT S |  |  |
| A0014 | STO C |  |  |
| A0015 | $\mathrm{x}^{2}$ |  |  |
| A0016 | RCL B |  |  |
| A0017 | $\mathrm{x}^{2}$ |  |  |
| A0018 | + |  |  |
| A0019 | RCL A |  |  |
| A0020 | $\mathrm{x}^{2}$ |  |  |
| A0021 | - |  |  |
| A0022 | RCL $\div$ B |  |  |
| A0023 | RCL $\div$ C |  |  |
| A0024 | 2 |  |  |
| A0025 | $\div$ |  |  |
| A0026 | ACOS |  |  |
| A0027 | $\rightarrow$ HMS |  |  |
| A0028 | ANGLE $1=$ |  | $\longrightarrow$ EQN RCL A RCL N etc. ENTER to end |
| A0029 | PSE |  |  |
| A0030 | STOP |  |  |
| A0031 | RCL C |  |  |
| A0032 | $\mathrm{x}^{2}$ |  |  |
| A0033 | RCL A |  |  |
| A0034 | $\mathrm{x}^{2}$ |  |  |
| A0035 | + |  |  |
| A0036 | RCL B |  |  |
| A0037 | $\mathrm{x}^{2}$ |  |  |

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| A0038 | - |  |
| :--- | :--- | :--- |
| A0039 | RCL $\div$ C |  |
| A0040 | RCL $\div$ A |  |
| A0041 | 2 |  |
| A0042 | $\div$ |  |
| A0043 | ACOS |  |
| A0044 | $\rightarrow$ HMS |  |
| A0045 | ANGLE 2 |  |
| A0046 | PSE |  |
| A0047 | STOP |  |
| A0048 | RCL B |  |
| A0049 | $x^{2}$ |  |
| A0050 | RCL A |  |
| A0051 | x $^{2}$ |  |
| A0052 | + |  |
| A0053 | RCL C |  |
| A0054 | x $^{2}$ |  |
| A0055 | - |  |
| A0056 | RCL $\div$ A |  |
| A0057 | RCL $\div$ B |  |
| A0058 | 2 |  |
| A0059 | $\div$ |  |
| A0060 | ACOS |  |
| A0061 | STO D |  |
| A0062 | $\rightarrow$ HMS |  |
| A0063 | ANGLE $3=$ |  |
| A0064 | PSE |  |
| A0065 | STOP |  |
| A0066 | RCL D |  |
| A0067 | SIN |  |
| A0068 | 2 |  |
| A0069 | $\div$ |  |
| A0070 | RCLx A |  |
| A0071 | RCLx B |  |
| A0072 | AREA $=$ |  |
| A0073 | PSE |  |
| A0074 | STOP |  |
| A0075 | CF 10 |  |
| A0076 | RTN |  |
|  |  |  |

## Label Used

Label A Length $=345 \quad$ Checksum $=3 D D 1$
Use the length ( $\mathrm{LN}=$ ) and Checksum $(\mathrm{CK}=)$ values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

## Storage Registers Used

A Side length 1
B Side length 2
C Side length 3
D Angle 3 in decimal degrees
S Temporary storage of input side length

## Notes

(1) Program for computing the three angles and area of a plane triangle, given the lengths of the three sides.
(2) Angles are displayed in HP notation, i.e., DDD.MMSS.
(3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
(4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.

## Theory

This program accepts the three side lengths of the triangle, then uses to cosine rule to compute the three angles. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number.


Side 3
The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to $180^{\circ}$.

## Running the Program

Press XEQ A
Screen shows ENTER SIDE 1 briefly, then prompts with S?
Enter the length of side 1 and press R/S.
Screen shows ENTER SIDE 2 briefly, then prompts with S?
Enter the length of side 2 and press R/S.
Screen shows ENTER SIDE 3 briefly, then prompts with S?
Enter the length of side 3 and press R/S.
Screen shows ANGLE 1 = briefly, then shows Angle 1 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows AREA $=$ briefly, then shows the area in the lower $(\mathrm{X})$ register.
Press R/S to end program. This clears Flag 10, which was set at the start of the program.

## Sample Computations

## Triangle 1

Inputs: $\quad$ Side Length $1=100.000$
Side Length $2=100.000$
Side Length $3=100.000$
Results: Angle $1=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $2=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $3=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Area $=4,330.127$
Check angle sum $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Triangle 2

Side Length $1=10.000$
Side Length $2=10.000$
Side Length $3=18.000$
Angle $1=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Angle $2=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Angle $3=128^{\circ} 18^{\prime} 58^{\prime \prime}$
Area $=39.230$
Check $=180^{\circ} 00^{\prime} 00^{\prime \prime}$
2. Given 2 sides and included angle, compute the other two angles, other side and the area

Programmer: Dr. Bill Hazelton
Date: December, 2005.

| Line | Instruction | Display | Program Entry Instructions |
| :---: | :---: | :---: | :---: |
| B0001 | LBL B |  |  |
| B0002 | SF 10 |  | $\cdots$ FLAGS SF . 0 |
| B0003 | ENTER SIDE 1 |  | $\rightarrow$ EQN RCLE RCL N etc. ENTER to end |
| B0004 | PSE |  |  |
| B0005 | INPUT S |  |  |
| B0006 | STO A |  |  |
| B0007 | ENTER SIDE 2 |  | $>$ EQN RCLE RCL N etc. ENTER to end |
| B0008 | PSE |  |  |
| B0009 | INPUT S |  |  |
| B0010 | STO B |  |  |
| B0011 | ENTER ANGLE 3 |  | $\xrightarrow[\square]{ } \rightarrow$ EN RCL E RCL N etc. ENTER to end |
| B0012 | PSE |  |  |
| B0013 | INPUT S |  |  |
| B0014 | $\rightarrow \mathrm{HR}$ |  |  |
| B0015 | STO C |  |  |
| B0016 | COS |  |  |
| B0017 | RCLx A |  |  |
| B0018 | RCLx B |  |  |
| B0019 | 2 |  |  |
| B0020 | x |  |  |
| B0021 | RCL A |  |  |
| B0022 | $\mathrm{x}^{2}$ |  |  |
| B0023 | $x<>y$ |  |  |
| B0024 | - |  |  |
| B0025 | RCL B |  |  |
| B0026 | $\mathrm{x}^{2}$ |  |  |
| B0027 | + |  |  |
| B0028 | $\sqrt{ } \mathrm{x}$ |  |  |
| B0029 | STO D |  |  |
| B0030 | SIDE 3 = |  | $\longrightarrow$ EQN RCL S RCL I etc. ENTER to end |
| B0031 | PSE |  |  |
| B0032 | STOP |  |  |
| B0033 | RCL C |  |  |
| B0034 | SIN |  |  |
| B0035 | RCL $\div$ D |  |  |
| B0036 | STO D |  |  |
| B0037 | RCLx A |  |  |
| B0038 | ASIN |  |  |
| B0039 | $\rightarrow$ HMS |  |  |

Plane Triangle Solutions


## Notes

(1) Program for computing two angles, one side and the area of a plane triangle, given the lengths of two sides and the angle between them.
(2) Angles are displayed in HP notation, i.e., DDD.MMSS, and should also be entered in that format.
(3) Whatever linear units are used (and they should be the same for all the sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
(4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.

## Storage Registers Used

A Side length 1
B Side length 2
C Angle 3 in decimal degrees
D Side 3 temporary storage, then sine ratio temporary storage
S Temporary input storage

## Label Used

Label B Length $=273 \quad$ Checksum $=927 \mathrm{C}$
Use the length ( $\mathrm{LN}=$ ) and Checksum ( $\mathrm{CK}=$ ) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

## Theory

This program accepts the two known side lengths of the triangle, followed by the included angle, then uses to cosine rule to compute the other side. The sine rule is used to compute the other two angles. The area is computed using the length of the two given sides and half the sine of the given angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. Side 1 is assumed to be to the right of the included known angle, here angle 3 . Side 2 is to the left of the included angle.


The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to $180^{\circ}$.

## Sample Computations

## Triangle 1

Inputs: $\quad$ Side Length $1=100.000$
Side Length $2=100.000$
Angle $3=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Results: $\quad$ Side $3=100.000$
Angle $1=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $2=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Area $=4,330.127$
Check angle sum $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Triangle 2

Side Length $1=10.000$
Side Length $2=10.000$
Angle $3=128^{\circ} 18^{\prime} 58^{\prime \prime}$
Side $3=18.000$
Angle $1=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Angle $2=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Area $=39.230$
Check $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Running the Program

Press XEQ B
Screen shows ENTER SIDE 1 briefly, then prompts with S?
Enter the length of side 1 and press R/S.
Screen shows ENTER SIDE 2 briefly, then prompts with S?
Enter the length of side 2 and press R/S.
Screen shows ENTER ANGLE 3 briefly, then prompts with S?
Enter angle 3 in HP notation and press R/S.
Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE $1=$ briefly, then shows Angle 1 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows AREA = briefly, then shows the area in the lower (X) register.
Press R/S to end program. This clears Flag 10, which was set at the start of the program.

## 3. Given 2 angles and the included side, compute the other angle, the other 2 sides and area

Programmer: Dr. Bill Hazelton
Date: December, 2005.

| Line | Instruction | Display | Program Entry Instructions |
| :---: | :---: | :---: | :---: |
| C0001 | LBL C |  |  |
| C0002 | SF 10 |  | $\rightarrow$ FLAGS SF . 0 |
| C0003 | ENTER ANGLE 1 |  | $\rightarrow$ EQN RCL E RCL N etc. ENTER to end |
| C0004 | PSE |  |  |
| C0005 | INPUT S |  |  |
| C0006 | $\rightarrow$ HR |  |  |
| C0007 | STO A |  |  |
| C0008 | ENTER ANGLE 2 |  | $r$ EQN RCLE RCL N etc. ENTER to end |
| C0009 | PSE |  |  |
| C0010 | INPUT S |  |  |
| C0011 | $\rightarrow$ HR |  |  |
| C0012 | STO B |  |  |
| C0013 | ENTER SIDE 3 |  | $\longrightarrow \mathrm{EQN}$ RCL E RCL N etc. ENTER to end |
| C0014 | PSE |  |  |
| C0015 | INPUT S |  |  |
| C0016 | STO C |  |  |
| C0017 | 180 |  |  |
| C0018 | RCL- A |  |  |
| C0019 | RCL-B |  |  |
| C0020 | STO D |  |  |
| C0021 | $\rightarrow$ HMS |  |  |
| C0022 | ANGLE $3=$ |  | $\stackrel{\square}{\square} \mathrm{EQN}$ RCL A RCL N etc. ENTER to end |
| C0023 | PSE |  |  |
| C0024 | STOP |  |  |
| C0025 | RCL C |  |  |
| C0026 | RCL D |  |  |
| C0027 | SIN |  |  |
| C0028 | STO E |  |  |
| C0029 | $\div$ |  |  |
| C0030 | STO F |  |  |
| C0031 | RCL A |  |  |
| C0032 | SIN |  |  |
| C0033 | x |  |  |
| C0034 | STOx E |  |  |
| C0035 | SIDE 1 = |  | $\rightarrow \mathrm{EQN}$ RCL S RCL I etc. ENTER to end |
| C0036 | PSE |  |  |
| C0037 | STOP |  |  |
| C0038 | RCL F |  |  |
| C0039 | RCL B |  |  |

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| C0040 | SIN |  |  |
| :--- | :--- | :--- | :--- |
| C0041 | x |  |  |

## Notes

(1) Program for computing two side, an angle and area of a plane triangle, given two angles and the length of the side between them.
(2) Angles are displayed in HP notation, i.e., DDD.MMSS. They should also be entered in this format.
(3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
(4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.
(5) This program is essentially a surveying 'intersection' problem, and can also be interpreted as the 'two missing sides' problem.

## Storage Registers Used

A Angle 1 in decimal degrees
B Angle 2 in decimal degrees
C Length of Side 3
D Angle 3 temporary storage, in decimal degrees
E Area accumulation variable
F Sine ratio temporary storage
S Temporary input storage

Plane Triangle Solutions

## Theory

This program accepts the two known angles of the triangle, then the length of the side between them. It then computes the remaining angle by subtracting the sum of the two known angles from $180^{\circ}$. The sine rule is used to compute the other two sides. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. It is assumes that Angle 1 and Angle 2 are known and entered in that order, and side 3 is the side between them.


The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to $180^{\circ}$.

## Sample Computations

## Triangle 1

Inputs: $\quad$ Angle $1=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $2=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Side Length $3=100.000$
Results: Angle $3=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Side $1=100.000$
Side $2=100.000$
Area $=4,330.127$
Check angle sum $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Triangle 2

Angle $1=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Angle $2=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Side Length $3=18.000$
Angle $3=128^{\circ} 18^{\prime} 58^{\prime \prime}$
Side $1=10.000$
Side $2=10.000$
Area $=39.230$
Check $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Label Used

Label $\mathbf{C} \quad$ Length $=252 \quad$ Checksum $=$ A47F
Use the length ( $\mathrm{LN}=$ ) and Checksum ( $\mathrm{CK}=$ ) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

## Running the Program

Press XEQ C
Screen shows ENTER ANGLE 1 briefly, then prompts with S?
Enter angle 1 in HP notation and press R/S.
Screen shows ENTER ANGLE 2 briefly, then prompts with S?
Enter angle 2 in HP notation and press R/S.
Screen shows ENTER SIDE 3 briefly, then prompts with S?
Enter the length of side 3 and press R/S.
Screen shows ANGLE 3 = briefly, then shows Angle 1 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows SIDE $1=$ briefly, then shows Side 1 in the lower (X) register.
Press R/S to continue.
Screen shows SIDE $2=$ briefly, then shows Side 2 in the lower (X) register.
Press R/S to continue.
Screen shows AREA $=$ briefly, then shows the area in the lower $(\mathrm{X})$ register.
Press R/S to end program. This clears Flag 10, which was set at the start of the program.

## 4. Given 2 angles and a non-included side, compute the other angle, the other two sides and area

Programmer: Dr. Bill Hazelton
Date: December, 2005.

| Line | Instruction | Display | Program Entry Instructions |
| :---: | :---: | :---: | :---: |
| D0001 | LBL D |  |  |
| D0002 | SF 10 |  | $\rightarrow$ FLAGS SF . 0 |
| D0003 | ENTER ANGLE 1 |  | $\rightarrow$ EQN RCL E RCL N etc. ENTER to end |
| D0004 | PSE |  |  |
| D0005 | INPUT S |  |  |
| D0006 | $\rightarrow$ HR |  |  |
| D0007 | STO A |  |  |
| D0008 | ENTER ANGLE 3 |  | $\rightarrow$ EQN RCL E RCL N etc. ENTER to end |
| D0009 | PSE |  |  |
| D0010 | INPUT S |  |  |
| D0011 | $\rightarrow \mathrm{HR}$ |  |  |
| D0012 | STO B |  |  |
| D0013 | ENTER SIDE 3 |  | $\longrightarrow$ EQN RCL E RCL N etc. ENTER to end |
| D0014 | PSE |  |  |
| D0015 | INPUT S |  |  |
| D0016 | STO C |  |  |
| D0017 | RCL B |  |  |
| D0018 | SIN |  |  |
| D0019 | $\div$ |  |  |
| D0020 | STO D |  |  |
| D0021 | RCL A |  |  |
| D0022 | SIN |  |  |
| D0023 | X |  |  |
| D0024 | SIDE 1 = |  | $\longrightarrow$ EQN RCL S RCL I etc. ENTER to end |
| D0025 | PSE |  |  |
| D0026 | STOP |  |  |
| D0027 | 180 |  |  |
| D0028 | RCL- A |  |  |
| D0029 | RCL-B |  |  |
| D0030 | STO E |  |  |
| D0031 | $\rightarrow$ HMS |  |  |
| D0032 | ANGLE 2 = |  | $\longrightarrow$ EQN RCL A RCL N etc. ENTER to end |
| D0033 | PSE |  |  |
| D0034 | STOP |  |  |
| D0035 | RCL E |  |  |
| D0036 | SIN |  |  |
| D0037 | RCLx D |  |  |
| D0038 | STO E |  |  |
| D0039 | SIDE 2 = |  | $\rightarrow$ EQN RCL S RCL I etc. ENTER to end |

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| D0040 | PSE |  |
| :--- | :--- | :--- | :--- |
| D0041 | STOP |  |

## Notes

(1) Program for computing two side, an angle and area of a plane triangle, given two angles and the length of a side not between them.
(2) Angles are displayed in HP notation, i.e., DDD.MMSS. They should also be entered in this format.
(3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
(4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.

## Storage Registers Used

A Angle 1 in decimal degrees
B Angle 3 in decimal degrees
C Length of Side 3
D Sine ratio temporary storage
E Angle 2 temporary storage, in decimal degrees, then side 2 temporary storage
S Temporary input storage

## Label Used

Label D Length $=252 \quad$ Checksum $=5939$
Use the length ( $\mathrm{LN}=$ ) and Checksum ( $\mathrm{CK}=$ ) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

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## Theory

This program accepts the two known angles of the triangle, then the length of a side not between them. It then computes the remaining angle by subtracting the sum of the two known angles from $180^{\circ}$. The sine rule is used to compute the other two sides. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. It is assumes that Angle 1 and Angle 3 are known and entered in that order, and side 3 is the known side.


The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to $180^{\circ}$.

## Sample Computations

## Triangle 1

Inputs: $\quad$ Angle $1=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $3=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Side Length $3=100.000$
Results: $\quad$ Side $1=100.000$
Angle $2=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Side $2=100.000$
Area $=4,330.127$
Check angle sum $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Triangle 2

Angle $1=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Angle $3=128^{\circ} 18^{\prime} 58^{\prime \prime}$
Side Length $3=18.000$
Side $1=10.000$
Angle $2=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Side $2=10.000$
Area $=39.230$
Check $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Running the Program

Press XEQ D
Screen shows ENTER ANGLE 1 briefly, then prompts with S?
Enter angle 1 in HP notation and press R/S.
Screen shows ENTER ANGLE 3 briefly, then prompts with S?
Enter angle 3 in HP notation and press R/S.
Screen shows ENTER SIDE 3 briefly, then prompts with S?
Enter the length of side 3 and press R/S.
Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE $1=$ briefly, then shows Angle 1 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows SIDE $2=$ briefly, then shows Side 2 in the lower (X) register.
Press R/S to continue.
Screen shows AREA = briefly, then shows the area in the lower (X) register.
Press R/S to end program. This clears Flag 10, which was set at the start of the program.

## 5. Given 2 sides and an angle not between them, compute the other two angles, the other side and area (two possible solutions)

Programmer: Dr. Bill Hazelton
Date: December, 2005.

| Line | Instruction | Display | Program Entry Instructions |
| :---: | :---: | :---: | :---: |
| E0001 | LBL E |  |  |
| E0002 | SF 10 |  | $\mapsto$ FLAGS SF . 0 |
| E0003 | ENTER SIDE 1 |  | $\longrightarrow$ EQN RCLE RCL N etc. ENTER to end |
| E0004 | PSE |  |  |
| E0005 | INPUT S |  |  |
| E0006 | STO A |  |  |
| E0007 | ENTER SIDE 2 |  | $\mapsto$ EQN RCL E RCL N etc. ENTER to end |
| E0008 | PSE |  |  |
| E0009 | INPUT S |  |  |
| E0010 | STO B |  |  |
| E0011 | ENTER ANGLE 1 |  | $>$ EQN RCLE RCL N etc. ENTER to end |
| E0012 | PSE |  |  |
| E0013 | INPUT S |  |  |
| E0014 | $\rightarrow$ HR |  |  |
| E0015 | STO C |  |  |
| E0016 | SIN |  |  |
| E0017 | RCL $\div$ A |  |  |
| E0018 | STO D |  |  |
| E0019 | RCLx B |  |  |
| E0020 | ASIN |  |  |
| E0021 | STO E |  | Angle 2, solution 1 |
| E0022 | 180 |  |  |
| E0023 | $\mathrm{x}<>\mathrm{y}$ |  |  |
| E0024 | - |  |  |
| E0025 | RCL- C |  |  |
| E0026 | STO F |  | Angle 3, solution 1 |
| E0027 | SIN |  |  |
| E0028 | RCLx A |  |  |
| E0029 | RCL C |  |  |
| E0030 | SIN |  |  |
| E0031 | $\div$ |  |  |
| E0032 | STO G |  | Side 3, solution 1 |
| E0033 | RCL F |  |  |
| E0034 | SIN |  |  |
| E0035 | RCLx A |  |  |
| E0036 | RCLx B |  |  |
| E0037 | 2 |  |  |
| E0038 | $\div$ |  |  |

HP-33S Calculator Program
Plane Triangle Solutions

| E0039 | STO H |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

Plane Triangle Solutions

| E0086 | $\rightarrow$ HMS | $\square$ EQN RCL A RCL N etc. ENTER to end |
| :---: | :---: | :---: |
| E0087 | STOP |  |
| E0088 | ANGLE 3 = |  |
| E0089 | PSE | $\mapsto$ EQN RCL S RCL I etc. ENTER to end |
| E0090 | RCL F |  |
| E0091 | $\rightarrow$ HMS |  |
| E0092 | STOP |  |
| E0093 | SIDE 3 = |  |
| E0094 | PSE |  |
| E0095 | RCL G | $>$ EQN RCL A RCL R etc. ENTER to end |
| E0096 | STOP |  |
| E0097 | AREA = |  |
| E0098 | PSE |  |
| E0099 | RCL H | $\xrightarrow[r]{ }$ FLAGS CF . 0 |
| E0100 | STOP |  |
| E0101 | CF 10 |  |
| E0102 | RTN |  |

## Label Used

Label $\mathbf{E} \quad$ Length $=487 \quad$ Checksum $=$ E72F
Use the length ( $\mathrm{LN}=$ ) and Checksum ( $\mathrm{CK}=$ ) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

## Notes

(1) Program for computing the two angles, a side and area of a plane triangle, given the lengths of two sides and an angle not between the two sides.
(2) Angles are displayed in HP notation, i.e., DDD.MMSS.
(3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.
(4) The purpose of the EQN entries in the program is to provide a prompt ahead of the input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the R/S key.
(5) There are two possible solutions to this triangle. Each solution is presented separately. Note that if one of the solutions is not physically possible (usually solution 2), the program may return an error.

## Storage Registers Used

A Side length 1
B Side length 2
C Angle 3
D Temporary storage for sine ratio
E Angle 2 results
F Angle 3 results
G $\quad$ Side 3 results
H Area results
S Temporary storage of input side length

## Theory

This program accepts two side lengths of the triangle and an angle not between them, then uses the sine rule to compute one of the other angles. The third angle is computed by subtracting the sum of the other angles from $180^{\circ}$. The remaining side is computed using the sine rule. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. In this case, Side 1 and Side 2 are known, along with Angle 1.


There are two possible solutions, depending upon the solutions to Angle 2. Because it is determined using the sine rule, and $\arcsin (x)$ can have multiple values, there is one solution where Angle 2 lies between $0^{\circ}$ and $90^{\circ}$, and a second where Angle 2 lies between $90^{\circ}$ and $180^{\circ}$. Both these solutions are computed. The results are presented in two groups, with suitable text prompts.

Solution 1 is based on Angle 2 being less than $90^{\circ}$. Solution 2 is based on Angle 2 being greater than $90^{\circ}$.

The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to $180^{\circ}$.

## Running the Program

Press XEQ E
Screen shows ENTER SIDE 1 briefly, then prompts with S?
Enter the length of side 1 and press R/S.
Screen shows ENTER SIDE 2 briefly, then prompts with S?
Enter the length of side 2 and press R/S.
Screen shows ENTER ANGLE 1 briefly, then prompts with S?
Enter angle 1 in HP notation and press R/S.
Screen shows SOLUTION 1 briefly.
Screen shows ANGLE $2=$ briefly, then shows Angle 2 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows SIDE $3=$ briefly, then shows Side 3 in the lower (X) register.
Press R/S to continue.
Screen shows AREA $=$ briefly, then shows the area in the lower $(\mathrm{X})$ register.
Press R/S to continue.
Screen shows SOLUTION 2 briefly.
Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.
Press R/S to continue.
Screen shows SIDE $3=$ briefly, then shows Side 3 in the lower (X) register.
Press R/S to continue.
Screen shows AREA = briefly, then shows the area in the lower (X) register.
Press R/S to end program. This clears Flag 10, which was set at the start of the program.

## Sample Computations

## Triangle 1

Inputs: $\quad$ Side Length $1=100.000$
Side Length $2=100.000$
Angle $1=60^{\circ} 00^{\prime} 00^{\prime \prime}$

## Results: Solution 1

Angle $2=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $3=60^{\circ} 00^{\prime} 00^{\prime \prime}$
Side Length $3=100.000$
Area $=4,330.127$
[Check angle sum $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Solution 2

Angle $2=120^{\circ} 00^{\prime} 00^{\prime \prime}$
Angle $3=0^{\circ} 00^{\prime} 00^{\prime \prime}$
Side Length $3=0.000$
Area $=0.000$
Check angle sum $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

## Triangle 2

Side Length $1=10.000$
Side Length $2=10.000$
Angle $1=25^{\circ} 50^{\prime} 31^{\prime \prime}$

Angle $2=25^{\circ} 50^{\prime} 31^{\prime \prime}$
Angle $3=128^{\circ} 18^{\prime} 58^{\prime \prime}$
Side Length $3=18.000$
Area $=39.230$
Check $=180^{\circ} 00^{\prime} 00^{\prime \prime}$ ]

Angle $2=154^{\circ} 09^{\prime} 29^{\prime \prime}$
Angle $3=0^{\circ} 00^{\prime} 00^{\prime \prime}$ (very small)
Side Length $3=0.000$
Area $=0.000$
Check $=180^{\circ} 00^{\prime} 00^{\prime \prime}$

Clearly, neither of the results for Solution 2 are particularly good solutions (despite being mathematically valid and correct), because of the zero angles involved. This, as well as negative angles, is one way to spot an unsuitable solution. However, it is possible to have two perfectly reasonable solutions, in which case you will need to look beyond the given data to decide which is the required solution.

