

**Lambert Conformal Conic Traverse Computations**

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Line	Instruction	Line	Instruction	Line	Instruction
L0001	LBL L	O0014	2	O0053	STO+ M
L0002	CL $\Sigma$	O0015	$\div$	O0054	STO K
L0003	INPUT A	O0016	STO+ V	O0055	STOP
L0004	INPUT E	O0017	111120	O0056	RCL L
L0005	INPUT N	O0018	RCL B	O0057	$\rightarrow$ HMS
L0006	INPUT R	O0019	$\sqrt{x}$	O0058	STOP
L0007	INPUT C	O0020	$\div$	O0059	RCL M
L0008	INPUT F	O0021	$\div$	O0060	6
L0009	INPUT B	O0022	STO+ L	O0061	$\div$
L0010	INPUT X	O0023	RCL I	O0062	STOP
L0011	INPUT Y	O0024	2	O0063	RCL $\times$ H
L0012	INPUT L	O0025	$\div$	O0064	STOP
L0013	RCL L	O0026	STO+ U	O0065	RCL V
L0014	$\rightarrow$ HR	O0027	XEQ G	O0066	RCL- J
L0015	STO L	O0028	XEQ K	O0067	RCL- N
L0016	STO O	O0029	4	O0068	RCL J
L0017	RCL X	O0030	$\times$	O0069	3
L0018	STO U	O0031	STO+ M	O0070	$\div$
L0019	RCL Y	O0032	RCL J	O0071	+
L0020	STO V	O0033	2	O0072	RCL $\times$ I
L0021	XEQ G	O0034	$\div$	O0073	2.54 E -9
L0022	$\rightarrow$ HMS	O0035	STO+ V	O0074	$\times$
L0023	STOP	O0036	111120	O0075	RCL B
L0024	XEQ K	O0037	RCL B	O0076	$\times$
L0025	STO M	O0038	$\sqrt{x}$	O0077	STOP
L0026	STOP	O0039	$\div$	O0078	RCL V
O0001	LBL O	O0040	$\div$	O0079	RCL- N
O0002	$x < 0 ?$	O0041	STO+ L	O0080	RCL J
O0003	GTO P	O0042	RCL I	O0081	3
O0004	$\rightarrow$ HR	O0043	2	O0082	$\div$
O0005	STOP	O0044	$\div$	O0083	-
O0006	STO H	O0045	STO+ U	O0084	RCL $\times$ I
O0007	$\theta, r \rightarrow y, x$	O0046	RCL U	O0085	-2.54 E -9
O0008	STO J	O0047	RCL V	O0086	$\times$
O0009	$x < > y$	O0048	STOP	O0087	RCL B
O0010	STO I	O0049	XEQ G	O0088	$\times$
O0011	$x < > y$	O0050	$\rightarrow$ HMS	O0089	STOP
O0012	$\Sigma+$	O0051	STOP	O0090	RCL K
O0013	RCL J	O0052	XEQ K	O0091	STO M

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Line	Instruction	Line	Instruction	Line	Instruction
O0092	x <> y	G0001	LBL G	K0001	LBL K
O0093	GTO O	G0002	RCL U	K0002	RCL P
P0001	LBL P	G0003	RCL- F	K0003	$\sqrt{x}$
P0002	$\Sigma y$	G0004	$x^2$	K0004	RCL $\times$ C
P0003	$\Sigma x$	G0005	STO P	K0005	RCL L
P0004	y,x $\rightarrow$ $\theta,r$	G0006	$\sqrt{x}$	K0006	COS
P0005	x <> y	G0007	RCL R	K0007	$\div$
P0006	$\rightarrow$ HMS	G0008	RCL- V	K0008	RCL $\div$ A
P0007	x <> y	G0009	$x^2$	K0009	1
P0008	STOP	G0010	STO+ P	K0010	RCL L
P0009	RCL L	G0011	$\sqrt{x}$	K0011	SIN
P0010	RCL O	G0012	$\div$	K0012	$x^2$
P0011	-	G0013	ATAN	K0013	RCL $\times$ E
P0012	$\rightarrow$ HMS	G0014	$\rightarrow$ HMS	K0014	-
P0013	STOP	G0015	RTN	K0015	$\sqrt{x}$
P0014	RCL U			P0016	$\times$
P0015	RCL- X			P0017	RTN
P0016	RCL V				
P0017	RCL- Y				
P0018	STOP				
P0019	XEQ C				
P0020	STO H				
P0021	STO I				
P0022	STO J				
P0023	STO K				
P0024	STO L				
P0025	STO M				
P0026	STO P				
P0027	STO U				
P0028	STO V				
P0029	STO X				
P0030	STO Y				
P0031	RTN				

**Notes**

- (1) This is a program to deal with a traverse that is to be brought onto the SPCS. The idea is to enter all the required data to compute the corrections to bring the data onto the SPCS, and do a preliminary traverse closure at the same time.
- (2) The program consists of three blocks of code. The first is the main program, which contains sub-blocks L, O and P. These need to be entered sequentially, as control runs from one to the next. Sub-block L contains the initial data entry part, sub-block O contains the main loop and computation part, and sub-block P contains the finishing and clean-up part.

There are two subroutines (actually functions), which can be located anywhere in program memory. They require certain values to be in memory registers and are

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called by the main program to compute specific values. Block G computes the grid convergence at the given point, as well as setting up some of the data for Block K. Block K computes the point scale factor at the given point.

- (3) The stack is used very little for data return. In most cases, a single value is presented in the X-register at a time. The only exceptions are when the co-ordinates of a point are presented, or the final misclosure information is shown.
- (4) Labeling is problematic on the HP-33S. Outputs are not labeled. Some inputs are labeled, but this is mostly the initial inputs. During the main part of the program there is no labeling, so you will have to follow the instructions fairly carefully.
- (5) The program will work properly in either SPCS 1983 or SPCS 1927, with the proviso that things may be computed slightly differently from SPCS 1927, owing to differences in the computations that produced the tables. If using SPCS 1927, use feet for the inputs and enter 0.092903 into B. This corrects the arc-to-chord correction computation to work in feet. Note that the tabulations of data supplied via the websites associated with this program have the 1927 data in meters.

Similarly you can work in feet (of whatever type) in SPCS 1983, provided you compute the correct value of  $a$ , the semi-major axis of the ellipse, and the other linear parameters for the zone, i.e.,  $N_0$ ,  $r_0$ ,  $E_0$ . But it is best to use meters for SPCS 1983.

- (6) Bearings and angles are entered and displayed in HP notation, i.e., DDD.MMSS
- (7) When entering the program, be aware that there are instructions STO P. These are not the STOP command!
- (8) The misclosure components in X (or E) and Y (or N) can be displayed by recalling  $\Sigma y$  and  $\Sigma x$  using the SUMS menu. (Note these are back-to-front.) Reversing their order using  $x \leftrightarrow y$  sets them up for conversion to polar. These can then be compared with the co-ordinate differences displayed as the misclosure components, which provide the output on misclosure components from the program.

**Theory**

The program works by running a small traverse closure routine and computing:

- the grid convergence ( $\gamma$ ) at each point;
- the point scale factor ( $k$ ) at each point and line midpoint;
- the co-ordinates of each point (E, N or X, Y);
- the latitude ( $\phi$ ) of each point;
- the line scale factor ( $K$ ) of each line;
- the grid length ( $L$ ) of each line; and
- the arc-to-chord corrections ( $\delta$ ) for each end of each line.

You enter all the data required to complete the above computations. Formulae are as per the SPCS Notes on the Lambert Conformal Conic Projection. Latitude is computed by dividing the  $\Delta N$  value of the line by 111120 to convert meters to degrees of latitude. This is corrected to feet if needed. Approximate latitudes are sufficient for computing the corrections for the traverse.

It is necessary to convert the bearings of the traverse to grid bearings before using them in the program. This will ensure that the co-ordinates and associated data are calculated correctly. After entering the grid co-ordinates of the first point, the grid convergence is displayed, and this value can be used to convert bearings on a true azimuth to an approximate grid azimuth. If you run the program only to this point, you can then stop and compute all the grid bearings, then restart the

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program. As all that has been done is data entry, these values should still be in place and can be accepted by pressing R/S quite quickly.

Whole circle bearings in HP notation are used. The program uses no error checking on entered data. Line scale factor is computed by combining the point scale factors of both ends and the middle of the line.

**Sample Computation**

For this computation, we will use the data from the SPCS tables that are currently available on the <http://homepage.mac.com/nwjh/HP-33S/> website. These are in an Excel spreadsheet or PDF files. Make sure that you are using the correct projection.

Using the example below on Ohio South Zone 1983, we first compute all the grid bearings, which is straightforward as the given line already has a grid bearing.

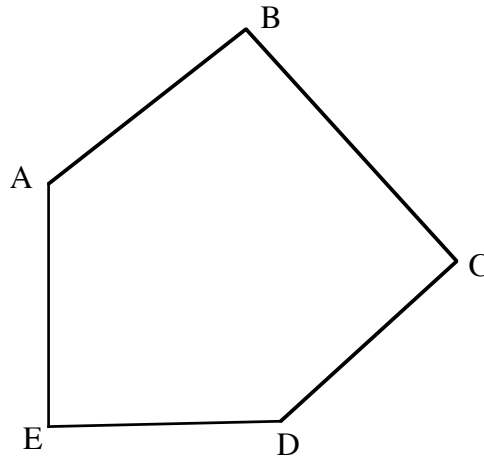
Lambert Conformal Conic Projection, SPCS 1983, Ohio Southern Zone.

Standard parallels	38° 44' N	40° 02' N	
Origin	38° 00' N	82° 30' W	
Co-ordinates of origin	E 600 000·000 (E <sub>0</sub> )	N 0·000 (in meters)	(false origin)

Point A has the following co-ordinates.

E 615 415·308	N 157 238·286	$\phi_A \approx 39^\circ 25' 00''$ N
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The grid bearing of the line from A to B is 47° 51' 27".



The interior angles at each traverse point, as observed in the field, are as follows:

A	132° 18' 25"
B	87° 23' 19"
C	85° 08' 08"
D	140° 54' 40"
E	94° 15' 30"

The lengths of the sides, reduced to ellipsoidal distances, are as follows:

AB	14 302·785
BC	12 821·076
CD	15 093·269
DE	6 394·974
EA	8 383·815

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Consulting the tables, the tabulated values for SPCS 1983 Ohio Southern Zone are:

$a = 6378\ 137\ \text{m}$                        $e^2 = 0.006\ 694\ 381$                        $N_0 = 153\ 682.09\ \text{m}$   
 $r_0 = 7032\ 869.04\ \text{m}$                        $n = 0.634\ 5195$                        $B = 1.0$  use for meters

Note that you don't need the F value from the tables for this program.

Begin by computing the grid bearings (azimuths) around the traverse. The bearings are:

AB      $47^\circ\ 51'\ 27''$   
 BC      $140^\circ\ 28'\ 08''$   
 CD      $235^\circ\ 20'\ 00''$   
 DE      $274^\circ\ 25'\ 20''$   
 EA      $0^\circ\ 09'\ 50''$

There is a 2" angular misclosure in the traverse, but we will ignore this for this example, especially as 1" of that is spherical excess that should be taken care of by the arc-to-chord corrections.

**Start the program** by pressing XEQ L

*You first enter data about the particular zone you are in.*

Prompt:	A?	Enter a	6378137	Press R/S
Prompt:	E?	Enter $e^2$	0.006694381	Press R/S
Prompt:	N?	Enter $N_0$	153682.09	Press R/S
Prompt:	R?	Enter $r_0$	7932869.04	Press R/S
Prompt:	C?	Enter n	0.6345195	Press R/S
Prompt:	F?	Enter $E_0$	600000	Press R/S
Prompt:	B?	Enter ft or m value	1 (for m)	Press R/S

*The next entries are about your starting point, A.*

Prompt:	X?	Enter E or X	615415.308	Press R/S
Prompt:	Y?	Enter N or Y	157238.286	Press R/S
Prompt:	L?	Enter latitude	39.25 (39° 25')	Press R/S

*The program now presents the grid convergence at A.*

0.03536                      ( $0^\circ\ 03'\ 53.6''$ )                      Press R/S

*The program now presents the point scale factor at A.*

0.9999419                      Don't press R/S yet!

You can use the grid convergence for the starting point to compute grid bearings for the traverse, if necessary, but outside this program.

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*Enter the data for side 1, A to B.* (This is the start of the main program loop)

Enter the bearing of the side: 47.5127 (47° 51' 27") Now press R/S

Enter the length of the side: 14302.785 Press R/S

*The program now presents the co-ordinates of point B in the Y and X registers of the stack.*

E 626020.513 (in the Y register)

N 166835.123 (in the X register) Press R/S

*The program now presents the grid convergence at B.*

$\gamma = 0.06472$  (0° 06' 47.2") Press R/S

*The program now presents the point scale factor at B.*

$k = 0.999\ 9464$  Press R/S

*The program now presents the latitude of point B.*

$\phi = 39.3011$  (30° 30' 11") Press R/S

*The program now presents the line scale factor for the line A to B.*

$K = 0.999\ 9438$  Press R/S

*The program now presents the grid distance for the line A to B.*

$L = 14301.981$  Press R/S

*The program now presents the arc-to-chord correction at A going to B*

$\delta_{AB} = +0.18$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at B going to A.*

$\delta_{BA} = -0.27$  (in seconds) Don't press R/S yet!

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*Enter the data for side 2, B to C.*

Enter the bearing of the side: 140.2808 (148° 28' 08") Now press R/S

Enter the length of the side: 12821.076 Press R/S

*The program now presents the co-ordinates of point C in the Y and X registers of the stack.*

E 634181.091 (in the Y register)

N 156946.495 (in the X register) Press R/S

*The program now presents the grid convergence at C.*

$\gamma = 0.0902$  (0° 09' 02") Press R/S

*The program now presents the point scale factor at C.*

$k = 0.999\ 9496$  Press R/S

*The program now presents the latitude of point C.*

$\phi = 39.2451$  (30° 24' 51") Press R/S

*The program now presents the line scale factor for the line B to C.*

$K = 0.999\ 9478$  Press R/S

*The program now presents the grid distance for the line B to C.*

$L = 12820.406$  Press R/S

*The program now presents the arc-to-chord correction at B going to C*

$\delta_{BC} = +0.20$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at C going to B.*

$\delta_{CB} = -0.14$  (in seconds) Don't press R/S yet!

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*Enter the data for side 3, C to D.*

Enter the bearing of the side: 235.2000 (235° 20' 00") Now press R/S

Enter the length of the side: 15093.269 Press R/S

*The program now presents the co-ordinates of point D in the Y and X registers of the stack.*

E 621767.253 (in the Y register)

N 148361.427 (in the X register) Press R/S

*The program now presents the grid convergence at D.*

$\gamma = 0.0537$  (0° 05' 37") Press R/S

*The program now presents the point scale factor at D.*

$k = 0.999\ 9451$  Press R/S

*The program now presents the latitude of point D.*

$\phi = 39.2012$  (30° 20' 12") Press R/S

*The program now presents the line scale factor for the line C to D.*

$K = 0.999\ 9451$  Press R/S

*The program now presents the grid distance for the line C to D.*

$L = 15092.469$  Press R/S

*The program now presents the arc-to-chord correction at C going to D.*

$\delta_{CD} = -0.01$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at D going to C.*

$\delta_{DC} = -0.08$  (in seconds) Don't press R/S yet!

[Note: the line C to D crosses the  $N_0$  line (the central parallel) and you can see that the arc-to-chord correction has the same sign at both ends of the line. The line has an S-shape, although the curvature is very small, as it is so close to  $N_0$ .]



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*Enter the data for side 4, D to E.*

Enter the bearing of the side: 274.2520 (274° 25' 20") Now press R/S

Enter the length of the side: 6394.974 Press R/S

*The program now presents the co-ordinates of point E in the Y and X registers of the stack.*

E 615391.318 (in the Y register)

N 148854.516 (in the X register) Press R/S

*The program now presents the grid convergence at E.*

$\gamma = 0.0353$  (0° 03' 53") Press R/S

*The program now presents the point scale factor at E.*

$k = 0.999\ 9430$  Press R/S

*The program now presents the latitude of point E.*

$\phi = 39.2028$  (30° 20' 28") Press R/S

*The program now presents the line scale factor for the line D to E.*

$K = 0.999\ 9440$  Press R/S

*The program now presents the grid distance for the line D to E.*

$L = 6394.616$  Press R/S

*The program now presents the arc-to-chord correction at D going to E.*

$\delta_{DE} = +0.08$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at E going to D.*

$\delta_{ED} = -0.08$  (in seconds) Don't press R/S yet!

[Note that the arc-to-chord corrections are more conventional, being equal and opposite (if small), because the line is predominantly east-west, i.e., no significant variation in distance from  $N_0$ .]

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*Enter the data for side 5, E to A.*

Enter the bearing of the side: 0.0950 (0° 09' 50") Now press R/S

Enter the length of the side: 8383.815 Press R/S

*The program now presents the co-ordinates of point A in the Y and X registers of the stack.*

E 615415.299 (in the Y register)  
 N 157238.297 (in the X register) Press R/S

*The program now presents the grid convergence at A.*

$\gamma = 0.03536$  (0° 03' 53.6") Press R/S

*The program now presents the point scale factor at A.*

$k = 0.999\ 9419$  Press R/S

*The program now presents the latitude of point A.*

$\phi = 39.2500$  (30° 25' 00") Press R/S

*The program now presents the line scale factor for the line E to A.*

$K = 0.999\ 9423$  Press R/S

*The program now presents the grid distance for the line E to A.*

$L = 8383.331$  Press R/S

*The program now presents the arc-to-chord correction at E going to A.*

$\delta_{EA} = 0.00$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at A going to E.*

$\delta_{AE} = 0.00$  (in seconds) Don't press R/S yet!

***Finish and clean up.***

As we are now finished the traverse, enter a negative bearing to jump out of the main loop.

Enter -1 Now Press R/S

*The program presents the misclosure in the X and Y registers of the stack.*

In Y register 311.12 (311° 12') Bearing of misclosure  
 In X register 0.014 Length of misclosure Press R/S

*The program now presents the misclosure in the latitude computation.*

0.00 Latitude misclosure in DDD.MMSS Press R/S

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The program now presents the misclosure components, computed by comparing co-ordinates.

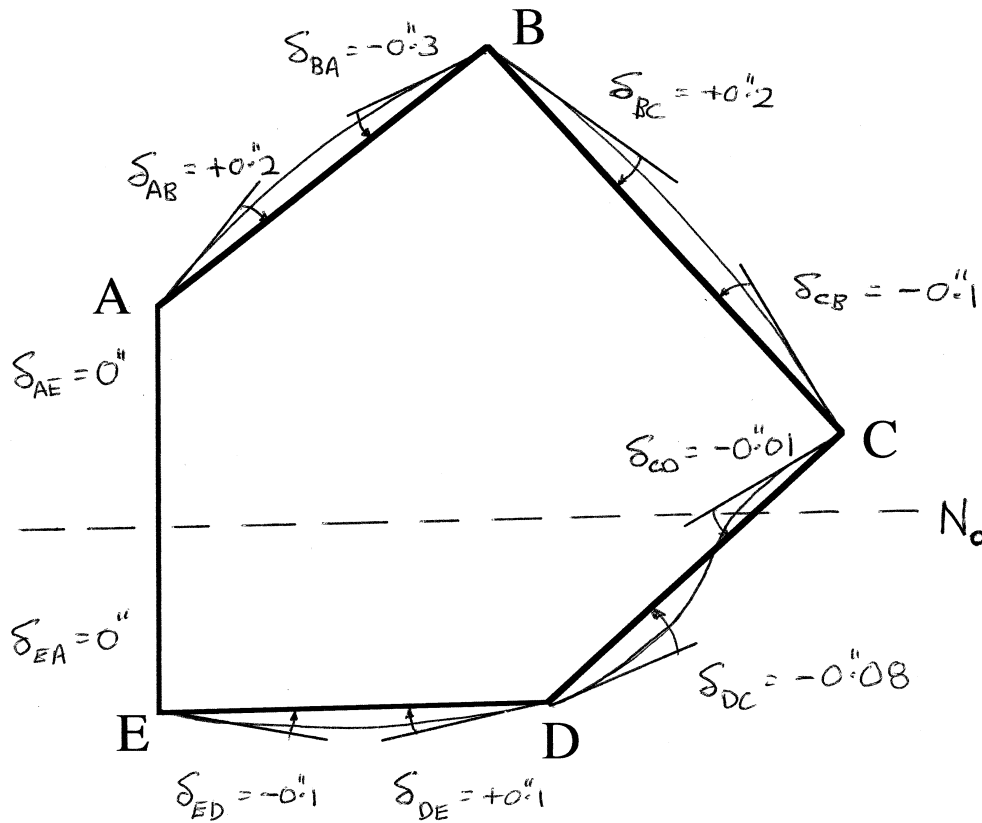
In Y register    -0.009             $\Delta E$   
 In X register    0.011                 $\Delta N$

At this point the program has completed all its required operations. If you press R/S once more, the program will clear the temporary variables (shown with an asterisk in the listing below) from the memory registers, but leave the main data, in case you want to recompute the traverse, or another in the same zone.

**Final Computations**

You now have the arc-to-chord corrections for the whole traverse, together with the point and line scale factors and the grid distances. If the arc-to-chord corrections are large enough to justify recomputing the bearings, do this manually. Then run the data (starting co-ordinates, grid bearings and grid distances) through any closure program to compute the final co-ordinates. By converting all your data to plane bearings and distances, all computations can be undertaken with plane computational methods.

The solution of the arc-to-chord corrections, showing the shape of the measured geodetic lines, as projected onto the Lambert map projection. Note the S-shaped nature of the line from C to D as it crosses the  $N_0$  line at an angle to north-south, while the line from E to A is north-south and has no curvature. Note also that the amount of curvature is very small.



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So, to compute the corrected grid bearings for the final adjustment, we get the following solution.

Grid Bearing B to A	227° 51' 27"	
	<u>+0.3"</u>	(chord-to-arc correction)
	227° 51' 27.3"	
Observed Angle @ B	<u>87° 23' 19"</u>	
	140° 28' 08.3"	
	<u>+0.2"</u>	(arc-to-chord correction)
Grid Bearing B to C	140° 28' 08.5"	
Grid Bearing C to B	320° 28' 08.5"	
	<u>+0.1"</u>	(chord-to-arc correction)
	320° 28' 08.6"	
Observed Angle @ C	<u>85° 08' 08"</u>	
	235° 20' 00.6"	
	<u>+0.01"</u>	(arc-to-chord correction)
Grid Bearing C to D	235° 20' 00.6"	
Grid Bearing D to C	55° 20' 00.6"	
	<u>+0.1"</u>	(chord-to-arc correction)
	55° 20' 00.7"	
Observed Angle @ D	<u>140° 54' 40"</u>	
	274° 25' 20.7"	
	<u>+0.1"</u>	(arc-to-chord correction)
Grid Bearing D to E	274° 25' 20.8"	
Grid Bearing E to D	94° 25' 20.8"	
	<u>+0.1"</u>	(chord-to-arc correction)
	94° 25' 20.9"	
Observed Angle @ E	<u>94° 15' 30"</u>	
	0° 09' 50.9"	
	<u>0.0"</u>	(arc-to-chord correction)
Grid Bearing E to A	0° 09' 50.9"	
Grid Bearing A to E	180° 09' 50.9"	
	<u>0.0"</u>	(chord-to-arc correction)
	180° 09' 50.9"	
Observed Angle @ A	<u>132° 18' 25"</u>	
	47° 51' 25.9"	
	<u>+0.2"</u>	(arc-to-chord correction)
Grid Bearing A to B	47° 51' 26.1"	

Comparing the closing bearing to the initial bearing, we see that the difference is 0.9". Since we computed the spherical excess for this traverse to be 1.1", and the overall angular misclosure to be 2", we can see that the arc-to-chord corrections have eliminated the spherical excess from the traverse.

The above grid bearings would now be used to compute the final co-ordinates, using the starting co-ordinates of point A and the grid distances computed during the program's run. These values would be used in an ordinary traverse closure program, e.g., Closure 2 for co-ordinates or Closure 1 for area within the polygon. Everything from here is done using plane methods. The traverse may also be adjusted at this stage, using a suitable method, e.g., least squares.

**Lambert Conformal Conic Traverse Computations****Storage Registers Used** (\* = temporary variable)

<b>A*</b>	a	semi-major axis of ellipsoid	<b>L</b>	$\phi$	latitude at current point
<b>B*</b>	1.0 or 0.0929	(feet conversion for 1927)	<b>M</b>	K	line scale factor
<b>C*</b>	n	the constant of the cone	<b>N*</b>	$N_0$	central parallel
<b>E*</b>	$e^2$	eccentricity of the ellipsoid	<b>O*</b>		latitude of starting point
<b>F*</b>	$E_0$ or $X_0$	the false easting	<b>P</b>	r	plot radius of point
<b>H</b>		Ground length of current line	<b>R*</b>	$r_0$	plot radius of origin
<b>I</b>		DE of current line	<b>U</b>		X or E of current point
<b>J</b>		DN of current line	<b>V</b>		Y or N of current point
<b>K</b>	k	point scale factor at current point	<b>X</b>		X or E of starting point
			<b>Y</b>		Y or N of starting point

Statistical Registers:  $\Sigma x$  = Current  $\Delta Y$  or  $\Delta N$  from starting point

$\Sigma y$  = Current  $\Delta X$  or  $\Delta E$  from starting point

**Labels Used**

Label **G**      Length = 45      Checksum = 5E1F

Label **K**      Length = 63      Checksum = 9BD2

Label **L**      Length = 78      Checksum = F8BF

Label **O**      Length = 423      Checksum = CC5A

Label **P**      Length = 93      Checksum = 4A44

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly.  
Use the sample computation to check proper operation after entry.