






















**Compute Values for a Parabolic Vertical Curve**

Programmer: Dr. Bill Hazelton

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Mnemonic: P for Parabolic Vertical Curve.

Line	Instruction	Display	User Instructions
P0001	LBL P		 LBL P
P0002	CLx		 CLEAR 1
P0003	ENTER		
P0004	ENTER		
P0005	ENTER		
P0006	SF 10		 FLAGS 1 .0
P0007	COMP V CURVE		 EQN RCL C, RCL O, etc.
P0008	PSE		 PSE
P0009	CLx		 CLEAR 1
P0010	STO S		
P0011	STO R		
P0012	STO Q		
P0013	STO P		
P0014	STO A		
P0015	STO L		
P0016	ENTER P I RD		 EQN RCL E, RCL N, etc.
P0017	PSE		 PSE
P0018	INPUT R	R?	 INPUT R
P0019	ENTER P I EL		 EQN RCL E, RCL N, etc.
P0020	PSE		 PSE
P0021	INPUT S	S?	 INPUT S
P0022	START GRADE		 EQN RCL S, RCL T, etc.
P0023	PSE		 PSE
P0024	INPUT P	P?	 INPUT P
P0025	END GRADE		 EQN RCL E, RCL N, etc.
P0026	PSE		 PSE
P0027	INPUT Q	Q?	 INPUT Q
P0028	RCL Q		
P0029	RCL— P		
P0030	STO A		
P0031	ENTER LENGTH		 EQN RCL E, RCL N, etc.
P0032	PSE		 PSE
P0033	INPUT L	L?	 INPUT L
P0034	RCL R		
P0035	RCL L		
P0036	2		
P0037	÷		
P0038	—		
P0039	STO U		
P0040	RCL S		

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**Curves 1**

Line	Instruction
P0041	RCL P
P0042	RCL× L
P0043	2
P0044	÷
P0045	—
P0046	STO V
P0047	RCL A
P0048	RCL÷ L
P0049	2
P0050	÷
P0051	STO B
P0052	RCL P
P0053	RCL÷ B
P0054	2
P0055	÷
P0056	+ / —
P0057	STO I
*****	Max/Min Pt. Data
P0058	MAX-MIN VALUES
P0059	PSE
P0060	RCL I
P0061	x <sup>2</sup>
P0062	RCL× B
P0063	RCL I
P0064	RCL× P
P0065	+
P0066	RCL+ V
P0067	STO E
P0068	MAX—MIN EL
P0069	PSE
P0070	VIEW E
P0071	MAX—MIN RD
P0072	PSE
P0073	RCL U
P0074	RCL+ I
P0075	STO D
P0076	VIEW D
*****	End Pts Data
P0077	END POINTS
P0078	PSE
P0079	START PT RD
P0080	PSE

Line	Instruction
P0081	VIEW U
P0082	START PT EL
P0083	PSE
P0084	VIEW V
P0085	END PT RD
P0086	PSE
P0087	RCL U
P0088	RCL+ L
P0089	STO F
P0090	VIEW F
P0091	RCL L
P0092	x <sup>2</sup>
P0093	RCL× B
P0094	RCL L
P0095	RCL× P
P0096	+
P0097	RCL+ V
P0098	STO E
P0099	END PT EL
P0100	PSE
P0101	VIEW E
*****	Get EL from RD
P0102	0
P0103	STO Y
P0104	STO X
P0105	COMP 1 EL (0-1)
P0106	PSE
P0107	INPUT Y
P0108	RCL Y
P0109	x ≤ 0?
P0110	GTO H
G0001	LBL G
G0002	ENTER RD
G0003	PSE
G0004	INPUT X
G0005	RCL X
G0006	RCL— U
G0007	STO I
G0008	x <sup>2</sup>
G0009	RCL× B
G0010	RCL I
G0011	RCL× P

Line	Instruction
G0012	+
G0013	RCL+ V
G0014	STO H
G0015	EL OF POINT
G0016	PSE
G0017	VIEW H
G0018	0
G0019	STO Y
G0020	AGAIN (0-1)
G0021	PSE
G0022	INPUT Y
G0023	RCL Y
G0024	x > 0?
G0025	GTO G
*****	Get RD from EL
H0001	LBL H
H0002	0
H0003	STO Y
H0004	STO H
H0005	STO X
H0006	COMP 1 RD (0-1)
H0007	PSE
H0008	INPUT Y
H0009	RCL Y
H0010	x ≤ 0?
H0011	GTO L
H0012	ENTER EL
H0013	PSE
H0014	INPUT H
H0015	RCL V
H0016	RCL— H
H0017	STO C
H0018	RCL P
H0019	x <sup>2</sup>
H0020	RCL B
H0021	RCL× C
H0022	4
H0023	×
H0024	—
H0025	STO D
H0026	x < 0?
H0027	THIS ELEV

\*\*\*\*\* These lines are simply comments in the code. You don't key it into the calculator!

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**Curves 1**

Line	Instruction
H0028	$x < 0?$
H0029	PSE
H0030	$x < 0?$
H0031	NOT ON CURVE
H0032	$x < 0?$
H0033	PSE
H0034	$x < 0?$
H0035	GTO L
H0036	RCL D
H0037	$\sqrt{x}$
H0038	STO D
H0039	RCL— P
H0040	RCL÷ B
H0041	2
H0042	÷
H0043	RCL+ U
H0044	STO X
H0045	FIRST RD
H0046	PSE
H0047	VIEW X
H0048	RCL D
H0049	+/-
H0050	RCL— P
H0051	RCL÷ B
H0052	2
H0053	÷
H0054	RCL+ U
H0055	STO X
H0056	SECOND RD
H0057	PSE
H0058	VIEW X
H0059	0
H0060	STO Y
H0061	AGAIN (0-1)
H0062	PSE
H0063	INPUT Y
H0064	RCL Y
H0065	$x > 0?$
H0066	GTO H
*****	Step thru RDs
L0001	LBL L
L0002	0
L0003	STO Y
L0004	STO X

Line	Instruction
L0005	STO H
L0006	STO D
L0007	STO C
L0008	STO I
L0009	STEP THRU RD
L0010	PSE
L0011	NO—YES (0-1)
L0012	PSE
L0013	INPUT Y
L0014	RCL Y
L0015	$x \leq 0?$
L0016	CF 10
L0017	$x \leq 0?$
L0018	RTN
L0019	FIRST INCRMNT
L0020	PSE
L0021	INPUT C
L0022	GENRL INCRMNT
L0023	PSE
L0024	INPUT D
L0025	FIRST PT
L0026	PSE
L0027	RD VALUE
L0028	PSE
L0029	VIEW U
L0030	EL VALUE
L0031	PSE
L0032	VIEW V
L0033	RD VALUE
L0034	PSE
L0035	RCL C
L0036	RCL+ U
L0037	STO X
L0038	VIEW X
L0039	RCL C
L0040	$x^2$
L0041	RCL× B
L0042	RCL C
L0043	RCL× P
L0044	+
L0045	RCL+ V
L0046	STO H
L0047	EL VALUE
L0048	PSE

Line	Instruction
L0049	VIEW H
L0050	RCL C
L0051	STO I
K0001	LBL K
K0002	RCL D
K0003	STO+ I
K0004	RCL I
K0005	RCL— L
K0006	$x \geq 0?$
K0007	GTO O
K0008	RCL I
K0009	$x^2$
K0010	RCL× B
K0011	RCL I
K0012	RCL× P
K0013	+
K0014	RCL+ V
K0015	STO H
K0016	RCL I
K0017	RCL+ U
K0018	STO X
K0019	RD VALUE
K0020	PSE
K0021	VIEW X
K0022	EL VALUE
K0023	PSE
K0024	VIEW H
K0025	GTO K
O0001	LBL O
O0002	END POINT
O0003	PSE
O0004	RCL L
O0005	$x^2$
O0006	RCL× B
O0007	RCL L
O0008	RCL× P
O0009	+
O0010	RCL+ V
O0011	STO H
O0012	RCL L
O0013	RCL+ U
O0014	STO X
O0015	RD VALUE
O0016	PSE

# HP-33S Calculator Program

## Compute Values for a Parabolic Vertical Curve

## Curves 1

Line	Instruction
O0017	VIEW X
O0018	EL VALUE
O0019	PSE
O0020	VIEW H
O0021	CF 10
O0022	RTN

### Notes

1. The \*\*\*\* lines are comments and are not to be entered into the calculator. They are there to make it easier to work through entering a long program.
2. The program offers options for the processing to be done at some steps, asking the user if a particular set of operations are to be done. The user is prompted for a yes/no answer, with the input variable being Y, which is set to 0 (no) by default. To skip the operation, just press R/S. To do the operation, key in 1, then press R/S. The sense is that 0 = no, 1 = yes. Note that there is a need to pause after the prompt, otherwise the entered value will not make it into the calculator, and the 'no' response will be acted upon.
3. Because grades are used, it is important that the units for elevation and running distance are the same. Otherwise the computations will not be correct.
4. Some of the calculations will yield results that are on the parabola being used, but not within the actual segment being used. This is particularly the case with finding maximum and minimum points, which may not lie within the limits of the curve, and finding running distances given elevations. The user should check that the running distances fall within the start and end points of the curve and ignore results that lie outside the curve. The program does not check these limits.

### Theory and Background

The theory of computing the various values for a parabolic 'equal-tangent-length' vertical curve is fairly straightforward. There are six basic possibilities of how the vertical curve can be placed between two grade lines, as shown in the figure overleaf. The curve allows the grade to be changed smoothly from the incoming grade (p) to the outgoing grade (q).

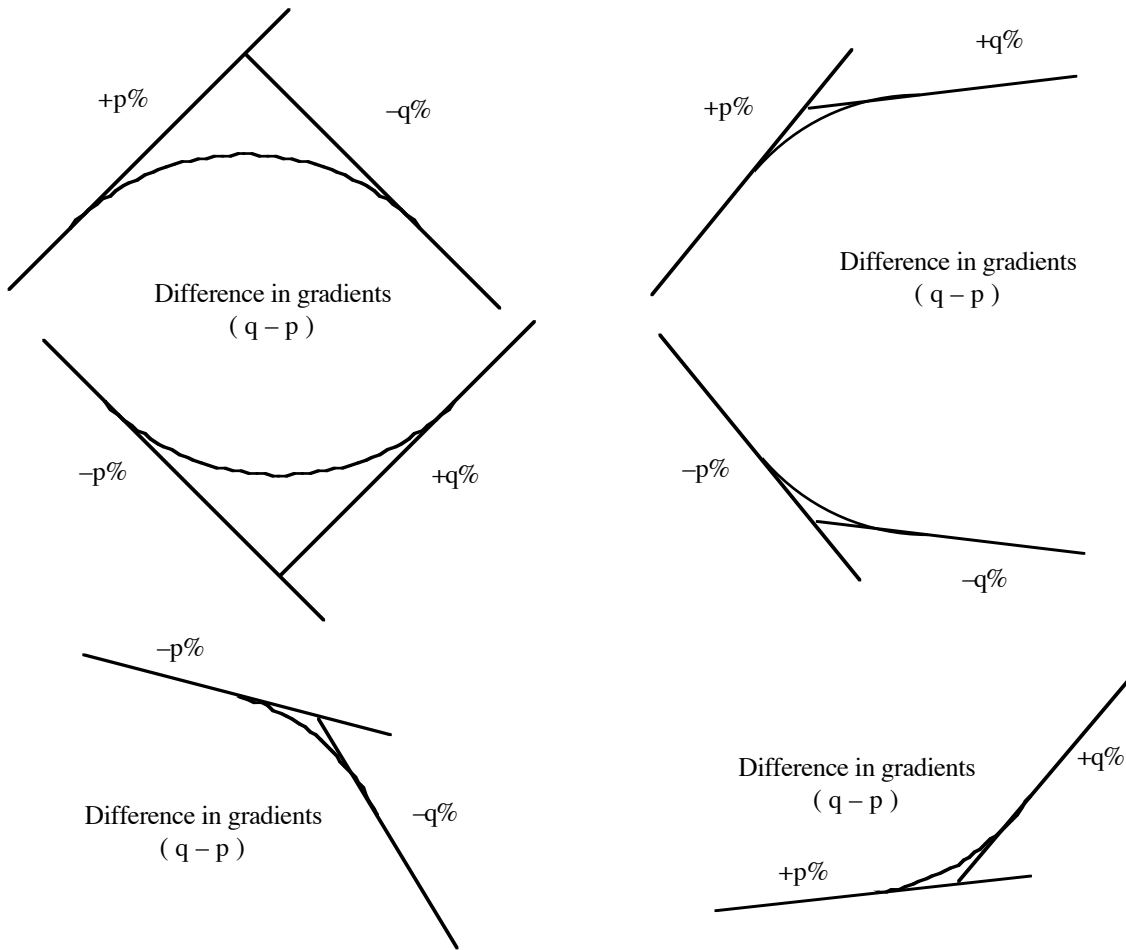
The convention with vertical curves is that they are drawn and computed going from left to right. If the grade is rising from left to right, the gradient is positive. If it is going down from left to right, the gradient is negative. The diagram overleaf shows the signs that the various gradients would take in the circumstances.

If the grades on opposite sides of the curve have the same sign, as in the two examples in the right in the figure above, there is no maximum or minimum value along the curve (other than at its end points). If the grades have opposite signs (as in the two examples on the left side of the above diagram), then there will be a maximum or minimum point of the parabola somewhere along the curve. In this case, the calculation of the elevation and location of such a point is meaningful. If such a calculation is made for the parabolas on the upper and central right and along the bottom in the diagram, the turning point for the parabola will be determined, but it will be well outside the

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part of the parabolic curve actually used. For example, the parabolas shown on the upper and central right side of the diagram above have their turning points well to the right of the end of the curve, while those along the bottom have their turning points well to the left of the actual curve.



For the purposes of vertical curve design, parabolas are the preferred curve. They are simpler to compute than circles or ellipses, but differ from them by amounts that are too small to matter in almost all cases. With a parabola, there is a constant change of grade (or gradient) around the curve, whereas for a circle there is a constant change of angle around the curve.

Another useful characteristic of the parabola is that it can be placed so that the lengths of the tangents are always equal. As the tangents are usually close to level (grades usually being fairly small), the horizontal distances from the point of intersection of the two grade lines to the tangent points are equal, and half the length of the curve. This allows easy placement of the curve with respect to the point of intersection of the grade lines. [Note that if the in and out grades are not equal and of opposite sign, the maximum or minimum point will not fall directly below the intersection point.]

### **Gradients, Grades, Slopes and Angles**

The slopes of the lines into and out of the vertical curve may be expressed in several ways. For this program, gradients should be entered as a decimal value of the gradient.

The gradient is the value of the change in elevation over a horizontal distance divided by that distance, i.e., rise over run, expressed as a decimal number. So, if the slope rises by 2 units for every 100 along, the gradient is  $+2 \div 100 = +0.02$ . If the slope falls by 5 units for 125 units along, the gradient is  $-5 \div 125 = -0.04$ .

The gradient can also be expressed as a percentage, which is simply the gradient value (as above) multiplied by 100 to convert it to a percentage. So the above examples would be +2% and -4%, respectively. Percentages are also handy in that they link in well to horizontal distances expressed in stations. As the distance is then in 100 ft units, a 1 foot rise would be a +1% grade, so the rise or fall over one 100 ft 'station' can be converted directly to the percentage gradient.

Gradients can also be expressed as a ratio of the rise to the run, and expressed in the form "1 in so many." To get this "so many" value, simply calculate the reciprocal value of the gradient (as a decimal), so for the two example given above, +0.02 would be  $\frac{1}{0.02} = 50$ , and so +1 in 50; -0.04 would be  $\frac{1}{0.04} = 25$ , and so -1 in 25.

Gradients can also be represented as the angle of the line from the horizontal, usually given in decimal degrees. The tangent of this angle will be the gradient, and the gradient can be converted to an angle by taking the arctangent of the gradient. So a gradient of +0.02 will give an angle of  $\arctan(+0.02) = +1.146^\circ$ , while -0.04 will give an angle of  $\arctan(-0.04) = -2.291^\circ$ . A slope of  $1^\circ$  would give a gradient of  $\tan(1^\circ) = 0.017$ .

Grade and gradient are used interchangeably, although sometimes they are applied to specific representations. For this program, convert all gradients to the decimal format, e.g., +0.02, -0.04. Be aware that the sign of the gradient is very important and *must* be included.

The term 'slope' is also used, but it usually doesn't refer to a specific representation.

### **Horizontal Distances**

Horizontal distances as used in the construction of linear objects are commonly expressed as a distance from a starting point somewhere along the object. How they are expressed depends upon the units being used, the country in which they are being used, and local practice. Similarly, what they are called also varies.

Distances in feet are commonly recorded as 'station' values. Here, it is assumed that a station is marked every 100 ft, and that the stations are numbered sequentially from the start, with distances on from the station noted as additional distance. So a distance of 12,546.78 ft would be recorded as 125 + 46.78, meaning 125 stations of 100 feet, plus 46.78 feet past that station. For many construction projects, having points every 100 ft (30.48 m) is very convenient, hence the popularity of this representation. It is easy to convert between the distance representation (12,546.78 ft) and the station representation (125 + 46.78): simply remove the + sign and place the digits together to go to distance, or open the digits two left of the decimal point and put in a +, to convert to stations. (Calculators prefer the distance version.)

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With metric units, 100 m is a bit long for station placement, so an equivalent metric representation never really caught on. In metric, it is more usual to use the distance representation (in meters) for all uses. This is also simpler as surveying moves away from the reliance on short lines for set-out (greatly aided by a station every 100 ft), to total station based set-out by co-ordinates across large areas.

The distances are known as 'stations' (when working with the 100 ft units), but this is a little odd when using a distance representation. In this case, the distance may be known as the 'Running Distance' (abbreviated RD), or the 'chainage' in some circumstances. As the station representation is so easily converted to the distance representation, and this program can also be used for metric applications, the horizontal distances in this program will be termed 'Running Distances' and often noted as RD.

### Calculating The Parabola

The general equation of a parabola is:

$$y = ax^2 + bx + c$$

The magnitude of the term  $a$  controls the sharpness of the parabola, while the sign of  $a$  controls the orientation. With  $a$  positive, the parabola is turned upwards and is bowl-shaped (the apex or turning point is the smallest  $y$  value), while with  $a$  negative, the parabola is hill-shaped, with the apex having the largest  $y$  value. So a summit or crest has a negative  $a$ , while a sag has a positive  $a$ .

It is convenient to use the starting point of the parabola (i.e., the first or left-most tangent point) as the origin of the co-ordinates. The elevation of this point (A on the diagram overleaf) above the chosen datum is equal to the term  $c$  in the equation. The slope  $\frac{dy}{dx}$  of any tangent is equal to  $2ax + b$ . But as at Point A,  $x = 0$ , the term  $b$  in the general equation is the slope or gradient at point A, the tangent gradient  $p$ .

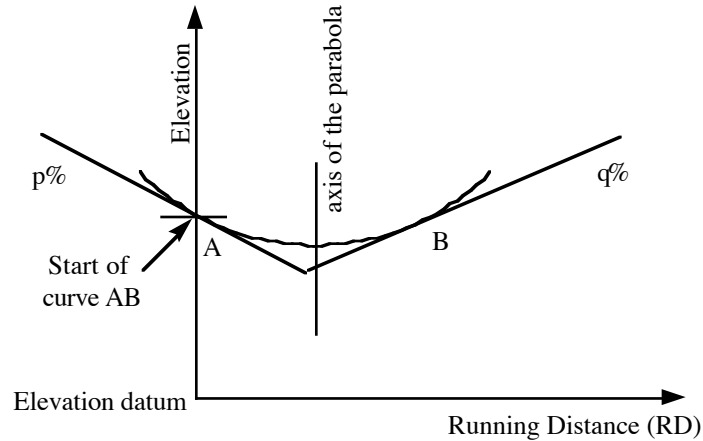
The second derivative  $\frac{d^2y}{dx^2}$  of the general parabola equation equals  $2a$ , a constant. This means that a tangent to a vertical-axis parabola changes a constant amount of *grade* for each increment of distance. (In contrast, a tangent to a vertical circular curve changes direction a constant amount of *angle* for equal increments of distance along the arc.) The useful consequence is that the rate of change of grade on a vertical curve is constant and equals  $2a$  per 100 units of distance (feet or meters, depending upon the units chosen). On a vertical curve the total change in direction between the profile grades is  $q - p$ , termed  $A$ .

If this change is accomplished on a curve  $L$  units long, the constant rate of change must be:

$$2a = \frac{q - p}{L} = \frac{A}{L}$$

with gradients  $q$ ,  $p$  and  $A$  in decimal form, and  $L$  in either feet or meters, consistent with the job.

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A practical formula for a general parabola for a vertical curve is therefore

$$\text{Elevation} = a (\text{RD} - \text{RD}_A)^2 + p (\text{RD} - \text{RD}_A) + \text{Elevation}_A$$

where RD is the running distance of any point,  $\text{RD}_A$  is the running distance of point A,  $a$  is the first parameter of the parabola (half the rate of change of grade as a decimal),  $p$  is the slope of the entry (or incoming) tangent (in decimal form), Elevation is the elevation of the point at running distance RD, and  $\text{Elevation}_A$  is the Elevation of point A. Elevation and RD should be in the same units, consistent with the entire job.

The location of the turning point of the parabola (the apex) can be computed by noting that the slope of the tangent at the turning point is 0, and solving  $2ax + b = 0$ . In this case, the location of the turning point is:  $x = \frac{-b}{2a}$ . This value can be converted to a running distance by adding the value of  $\text{RD}_A$ , and then used in the equation to compute the elevation of the maximum or minimum point on the curve.

Given the Elevation of a point on the curve, its location,  $x$ , from Point A can be computed by solving the equation:

$$a x^2 + p x + (\text{Elevation}_A - \text{Elevation}) = 0$$

As this is a quadratic, the standard quadratic solution will produce two solutions (in most cases), and it is up to the user to decide which is the most applicable. The program deals with the various cases (0, 1 and 2 solutions) separately. If there is a single solution, the apex (or nadir) of the curve has been selected. If there are no solutions, then the elevation selected is beyond the turning point elevation of the curve and therefore cannot lie on the curve.

For determining the elevations at set distances along the curve, the user can specify an initial increment, to bring the steps onto an even running distance, then specify a general increment, which will be used for the remainder of the curve. The program calculates the elevation of the first point on the curve (the left-hand tangent point), then moves along the initial increment, then proceeds along the curve using steps of the general increment, until the end of the curve is reached. The final point on the curve (the right-hand tangent point) is calculated and the program ends. For each point, the running distance and elevation are shown.



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## Curves 1

Note that if the final point calculated by increments happens to also be the end point, it will be calculated and displayed, and then the end point will be calculated again and displayed. This is because the test for coming to the end of the curve is that the increment is beyond the end point.

### Running the Program

Key in XEQ P. The program starts and displays:

COMP V CURVE

then prompts for the running distance of the intersection point of the two grade lines, displaying:

ENTER P I RD

then stops while displaying:

R?  
0.0000

Key in the RD of the intersection point and press R/S. The calculator then prompts for the elevation of the intersection point of the two grade lines, displaying:

ENTER P I EL

then stops while displaying:

S?  
0.0000

Key in the elevation of the intersection point, then press R/S. The calculator then prompts for the starting grade (p), displaying:

START GRADE

then stops while displaying:

P?  
0.0000

The value of the slope coming in to the vertical curve should be entered as a decimal value, e.g., 0.025 (an upward 2.5% gradient), then press R/S. The calculator then prompts for the grade coming out of the vertical curve, displaying:

END GRADE

then stops while displaying:

Q?  
0.0000

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**Curves 1**

Key in the value of the slope of the grade coming out of the curve, as a decimal grade, e.g., -0.04 (a downward 4% gradient), then press R/S. The calculator then prompts for the length of the curve, briefly displaying:

ENTER LENGTH

then stops while displaying:

L?  
0.0000

Key in the length of the curve desired and press R/S.

The calculator displays RUNNING briefly, and briefly displays:

MAX-MIN VALUES

then briefly displays:

MAX-MIN EL

then stops and displays the Elevation of the maximum or minimum point, looking as follows:

E=  
114.5611

Press R/S to continue. The calculator then briefly displays:

MAX-MIN RD

then stops and displays the Running Distance of the maximum or minimum point, looking as follows:

D=  
24,589.2222

Press R/S to continue.

This option computes the running distance and elevation of the end points of the curve, where the parabolic curve joins the tangent gradients. The calculator briefly displays:

END POINTS

then briefly displays:

START PT RD

then stops and display the running distance of the start point of the curve, such as follows:

U=  
24,367.0000

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**Curves 1**

Press R/S to continue. The calculator briefly displays:

START PT EL

then stops and displays the elevation of the start point of the curve, such as follows:

V=  
103.4500

Press R/S to continue. The calculator briefly displays:

END PT RD

then stops and display the running distance of the end point of the curve, such as follows:

F=  
24,767.0000

Press R/S to continue. The calculator briefly displays:

END PT EL

then stops and displays the elevation of the end point of the curve, such as follows:

E=  
107.4500

Press R/S to continue.

**A. *Compute Elevation at a Specified Running Distance***

With this option, the user can enter any running distance and the calculator will compute the elevation on the curve at that point. The calculator does not check if the running distance is on the curve segment actually being used, so the user must check this. The calculator briefly displays:

COMP 1 EL (0-1)

although the right-hand parenthesis will be off screen. The calculator then stops and displays:

Y?  
0.0000

If you don't want to run this option, press R/S and the program will advance to the next option. If you do want to run this option, key in 1 (or any number greater than 0), which signifies 'yes,' and press R/S. The calculator briefly displays:

ENTER RD

then stops and prompts for the RD to be entered, displaying:

X?  
0.0000

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**Curves 1**

Key in the running distance of the point of interest and press R/S. The calculator then briefly displays:

EL OF POINT

then stops and displays the elevation value of the selected point, such as follows:

H=  
114.3690

Press R/S to continue, and the calculator prompts to see if you want to compute another elevation, briefly displaying:

AGAIN (0-1)

then stopping and displaying:

Y?  
0.0000

If you want to do another point, key in 1 and press R/S. The program then prompts for the running distance (as above) and loops through the option until you decide not to do it again. If you don't want to do this option, press R/S, and the option ends.

**B. Compute the Running Distances at a Specified Elevation**

This option allows the user to enter an elevation and compute the running distance(s) at which it occurs. Since the curve is a parabola, there will be zero, one or two solutions. If the elevation cannot occur on the curve, i.e., the elevation is beyond the elevation of the turning point of the parabola, there will be zero answers. If the elevation chosen is that of the turning point, there will be just one answer, which will be given twice. The other results have two answers.

The calculator briefly displays:

COMP 1 RD (0-1)

with the right-hand parenthesis off the screen. The calculator then stops and displays the prompt:

Y?  
0.0000

If you want to skip this option, just press R/S and the calculator moves on to the next option. If you do want to run this option, key in 1 and press R/S. The calculator briefly displays:

ENTER EL

then stops and displays:

H?  
0.0000

Key in the elevation of interest, and press R/S.

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**Curves 1**

If the elevation is not on the curve, the calculator displays:

THIS ELEV

press R/S to continue, and the calculator displays:

NOT ON CURVE

Press R/S again, and you are taken to the next option.

If the elevation is elsewhere on the curve, the calculator briefly displays:

FIRST RD

then stops and displays the running distance of the first solution point, such as follows:

X=  
24,383.0819

Press R/S and the calculator briefly displays:

SECOND RD

then stops and displays the running distance of the second solution point, such as follows:

X=  
24,795.3625

It is up to the user to decide if the points fall within the end points of the curve, and chose points that are useful.

Press R/S to continue, and the calculator then briefly displays:

AGAIN (0-1)

then stops and displays:

Y?  
0.0000

To run the option again, key in and 1 and press R/S. The calculator will then return to the beginning of the options and prompt to see if you want to run the option, displaying briefly:

COMP 1 RD (0-1)

with the right-hand parenthesis off the screen. The calculator then stops and displays the prompt:

Y?  
0.0000

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**Compute Values for a Parabolic Vertical Curve**

**Curves 1**

Key in 1 to continue with the option. (Yes, I could have cut this out, but it would have taken another label, and there are already 6 in this program! If you add the issue of fixing the problem of an elevation not on the curve, there is another label!)

The calculator then prompts for the elevation to be entered, as above, and run through the option again. If you don't want to run the option again, press R/S and the option ends.

**C. Step Through a Series of Running Distances to get Elevations at each**

The final option allows the user to step through a series of equally-spaced points along the curve. As the start point is often at an odd running distance, this option allows the user to select a first increment, to allow the running distances to be brought to even values (e.g., exactly onto 100 ft stations), and then select a general increment to be applied successively until the end point is reached. The end point is calculated as the last point along the curve.

The option begins by displaying briefly:

STEP THRU RD

then displaying briefly:

NO-YES (0-1)

then stopping and displaying:

Y?  
0.0000

If you want to run this option, key in 1 and press R/S. If not, just press R/S and the program ends. If you are running the option, the calculator briefly displays:

FIRST INCRMNT

then stops and displays:

C=  
0.0000

Key in the first increment, then press R/S. The calculator briefly displays:

GENRL INCRMNT

then stops and displays:

D=  
0.0000

Key in the increment to be used for all the other distances, then press R/S. The calculator then briefly displays:

FIRST PT

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RD VALUE

then stops and displays the running distance of the start point, such as follows:

U=  
24,367.0000

Press R/S to continue. The calculator briefly displays:

EL VALUE

then stops and displays the elevation at the start point, such as follows:

V=  
103.4500

Press R/S to continue. The calculator then loops through the following sequence, briefly displaying:

RD VALUE

then stopping and displaying the next running distance value, such as follows:

X=  
24,467.0000

Press R/S to continue. The calculator briefly displays:

EL VALUE

then stops and displays the elevation value at the running distance just given, such as follows:

H=  
111.2000

Press R/S to continue through this loop until the increments extend past the last point. At this stage, the last point is displayed. The calculator briefly displays:

END POINT

then briefly displays:

RD VALUE

then stops and displays the running distance of the end point, such as follows:

X=  
24,767.0000

Press R/S to continue. The calculator briefly displays:

EL VALUE

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then stops and displays the elevation of the end point, such as follows:

$$H = 107.45$$

Press R/S to continue. The calculator briefly displays

PROGRAM END

and then stops at the end of the program, having reset Flag 10.

**Sample Computations**

The sample computations are based on the following general data:

Running Distance of the Point of Intersection = 24,567.0000

Elevation of the Point of Intersection = 123.4500

Length of the Curve = 400.0000

The running distance of the start point is therefore 24,367 and of the end point is 24,767.

The following tabulations show the results for different in and out gradients. The increments used for the running distance values are both 50 (both the first increment and the general increment).

Case	In Grade	Out Grade	Start EL	End EL	Max/Min RD	Max/Min EL
1	0.0200	-0.0200	119.45	119.45	24567.00	121.45
2	0.0400	-0.0600	115.45	111.45	24527.00	118.65
3	0.0600	-0.0400	111.45	115.45	24607.00	118.65
4	0.0800	-0.1000	107.45	103.45	24544.78	114.56
5	0.1000	-0.0800	103.45	107.45	24589.22	114.56
6	-0.0200	0.0400	127.45	131.45	24500.33	126.12
7	-0.0400	0.0200	131.45	127.45	24633.67	126.12
8	-0.0600	0.0800	135.45	139.45	24538.43	130.31
9	-0.0800	0.1000	139.45	143.45	24544.78	132.34
10	-0.1000	0.0600	143.45	135.45	24617.00	130.95
11	0.0200	0.0400	119.45	131.45	23967.00	115.45
12	0.0400	0.0200	115.45	127.45	25167.00	131.45
13	0.0600	0.0800	111.45	139.45	23167.00	75.45
14	0.0800	0.1000	107.45	143.45	22767.00	43.45
15	0.1000	0.0600	103.45	135.45	25367.00	153.45
16	-0.0200	-0.0400	127.45	115.45	23967.00	131.45
17	-0.0400	-0.0600	131.45	111.45	23567.00	147.45
18	-0.0600	-0.0400	135.45	115.45	25567.00	99.45
19	-0.0800	-0.1000	139.45	103.45	22767.00	203.45
20	-0.1000	-0.0800	143.45	107.45	26367.00	43.45



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**Curves 1**

Case	Running Distance								
	24367	24417	24467	24517	24567	24617	24667	24717	24767
1	119.45	120.33	120.95	121.33	121.45	121.33	120.95	120.33	119.45
2	115.45	117.14	118.20	118.64	118.45	117.64	116.20	114.14	111.45
3	111.45	114.14	116.20	117.64	118.45	118.64	118.20	117.14	115.45
4	107.45	110.89	113.20	114.39	114.45	113.39	111.20	107.89	103.45
5	103.45	107.89	111.20	113.39	114.45	114.39	113.20	110.89	107.45
6	127.45	126.64	126.20	126.14	126.45	127.14	128.20	129.64	131.45
7	131.45	129.64	128.20	127.14	126.45	126.14	126.20	126.64	127.45
8	135.45	132.89	131.20	130.39	130.45	131.39	133.20	135.89	139.45
9	139.45	136.01	133.70	132.51	132.45	133.51	135.70	139.01	143.45
10	143.45	138.95	135.45	132.95	131.45	130.95	131.45	132.95	135.45
11	119.45	120.51	121.70	123.01	124.45	126.01	127.70	129.51	131.45
12	115.45	117.39	119.20	120.89	122.45	123.89	125.20	126.39	127.45
13	111.45	114.51	117.70	121.01	124.45	128.01	131.70	135.51	139.45
14	107.45	111.51	115.70	120.01	124.45	129.01	133.70	138.51	143.45
15	103.45	108.33	112.95	117.33	121.45	125.33	128.95	132.33	135.45
16	127.45	126.39	125.20	123.89	122.45	120.89	119.20	117.39	115.45
17	131.45	129.39	127.20	124.89	122.45	119.89	117.20	114.39	111.45
18	135.45	132.51	129.70	127.01	124.45	122.01	119.70	117.51	115.45
19	139.45	135.39	131.20	126.89	122.45	117.89	113.20	108.39	103.45
20	143.45	138.51	133.70	129.01	124.45	120.01	115.70	111.51	107.45

These values should allow the program to be tested to make sure it is working properly. The above tabulated values were calculated by spreadsheet, rather than the calculator, but the calculator results were checked against these tabulations.

**Storage Registers Used**

- A**     Difference between the incoming and outgoing gradients.
- B**     Parameter  $a$  in the parabola equation.
- C**     Elevation difference. First increment value for ‘stepping’ option.
- D**     Running Distance of a computed point. General increment for ‘stepping’ option.
- E**     Elevation of a computed point.
- F**     Running Distance of a computed point.
- H**     Elevation of a computed point, and entered elevation to have RD calculated.
- I**     Distance along curve from start point.
- L**     Length of the curve to be computed.
- P**     Gradient of the incoming tangent (start grade)

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**Curves 1**

- Q** Gradient of the outgoing tangent (end grade).
- R** Running Distance of the Point of Intersection.
- S** Elevation of the Point of Intersection.
- U** Running Distance of the start point.
- V** Elevation of the start point.
- X** Running Distance entered to compute elevation at that point.
- Y** Yes/No variable for option choices.

**Statistical Registers:** Not used.

**Labels Used**

Label <b>P</b>	Length = 557	Checksum = 5A1D
Label <b>G</b>	Length = 117	Checksum = 5CA7
Label <b>H</b>	Length = 330	Checksum = 2187
Label <b>L</b>	Length = 255	Checksum = 3C67
Label <b>K</b>	Length = 91	Checksum = 1C67
Label <b>O</b>	Length = 91	Checksum = D6ED

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly.  
Use the sample computation to check proper operation after entry.

**Flags Used**

Flag 10 is used by this program. Flag 10 is set for this program, so that equations can be shown as prompts. At the end of the program, Flag 10 is cleared.