# "Missing Azimuth of One Side and Missing Distance of Another Side" Calculation 

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Date: June, 2007.


| Line | Instruction | Display | User Notes |
| :---: | :---: | :---: | :---: |
| N0039 | $x \geq y$ ? |  |  |
| N0040 | + |  |  |
| N0041 | $\mathrm{x}<\mathrm{y}$ ? |  |  |
| N0042 | - |  |  |
| N0043 | $\rightarrow$ HMS |  |  |
| N0044 | STO Q |  |  |
| N0045 | SLN 1 AZ |  | $\leftrightarrow$ EQN RCL S, RCL L, RCL N, etc. |
| N0046 | PSE |  |  |
| N0047 | VIEW Q |  | Solution 1, Azimuth result displayed |
| N0048 | R $\downarrow$ |  |  |
| N0049 | STO L |  |  |
| N0050 | SLN 2 LENG |  | $\leftrightarrow$ EQN RCL S, RCL L, RCL N, etc. |
| N0051 | PSE |  |  |
| N0052 | VIEW L |  | Solution 2, Length result displayed |
| N0053 | RCL A |  |  |
| N0054 | RCL B |  |  |
| N0055 | + |  |  |
| N0056 | $\rightarrow$ HMS |  |  |
| N0057 | STO Q |  |  |
| N0058 | SLN 2 AZ |  | $\mapsto$ EQN RCL S, RCL L, RCL N, etc. |
| N0059 | PSE |  |  |
| N0060 | VIEW Q |  | Solution 2, Azimuth result displayed |
| N0061 | CF 10 |  | (Enter using $\upharpoonright$ FLAGS CF .0) |
| N0062 | 0 |  |  |
| N0063 | STO A |  |  |
| N0064 | STO B |  |  |
| N0065 | STO C |  |  |
| N0066 | STO D |  |  |
| N0067 | STO L |  |  |
| N0068 | STO Q |  |  |
| N0069 | RTN |  |  |

## Notes

(1) Enter all the known sides of the traverse using the program stored under A, i.e., the closure program with area (Closure 1). The order in which the sides are entered doesn't matter.
(2) When all known sides have been entered and processed, enter the azimuth of the line missing the distance, then press XEQ N. This will take you to the start of the Closure 7 program.
(3) Azimuths are entered and displayed in HP notation, i.e., DDD.MMSS
(4) Several memory registers are using during computation (A, B, C, D, L, Q). These are cleared at the end of the program.
(5) There are almost always two solutions to this problem. The program calculates both solutions and displays them in turn. The user must decide which is the required solution for the task at hand.

## Theory

Once all the known sides have been entered (order does not matter), the resultant vector is known. This forms one side of a triangle, with the two unknown lines forming the other two sides. We know the length of the resultant vector, and the azimuths of two sides. So we can deduce the remaining data in the triangle.


Using the above situation as an example, line AC is the resultant vector from the vector addition of the sides AB and AC . This vector is $90^{\circ} 25^{\prime} 53^{\prime \prime}$ for 153.204 . The triangle to be solved forms as below. Note that the $\mathrm{AD}^{\prime} \mathrm{D}^{\prime}$ triangle is isosceles.


Using the sine rule to solve for the angle at D , the following occurs:

$$
\begin{aligned}
& \frac{\sin 14^{\circ} 10^{\prime} 03^{\prime \prime}}{57.23}=\frac{\sin D}{153.204} \\
& \sin D=\frac{153.204 \cdot \sin 14^{\circ} 10^{\prime} 03^{\prime \prime}}{57.23}=0.655212646
\end{aligned}
$$

## Missing Azimuth and Distance Calculation

Because the sine function has two angles between $0^{\circ}$ and $180^{\circ}$ that give the value 0.655212646 , and both can occur in a triangle, both must be resolved.

$$
D=40^{\circ} 56^{\prime} 09^{\prime \prime}\left(D^{\prime}\right) \text { and } 139^{\circ} 03^{\prime} 51^{\prime \prime}\left(D^{\prime \prime}\right)
$$

Since each possibility is equally valid mathematically, both must be solved. Solving for the angle at $A$ first, the two possibilities are:

$$
\begin{aligned}
& A^{\prime}=180^{\circ}-\left(14^{\circ} 10^{\prime} 03^{\prime \prime}+40^{\circ} 56^{\prime} 09^{\prime \prime}\right)=124^{\circ} 53^{\prime} 48^{\prime \prime} \text { and } \\
& A^{\prime \prime}=180^{\circ}-\left(14^{\circ} 10^{\prime} 03^{\prime \prime}+139^{\circ} 03^{\prime} 51^{\prime \prime}\right)=26^{\circ} 46^{\prime} 06^{\prime \prime}
\end{aligned}
$$

Solve for each possibility of the $C D$ line length using the sine rule, thus:

$$
\begin{aligned}
& \frac{57.23}{\sin 14^{\circ} 10^{\prime} 03^{\prime \prime}}=\frac{C D^{\prime}}{\sin 124^{\circ} 53^{\prime} 48^{\prime \prime}} \\
& C D^{\prime}=\frac{57.23 \cdot \sin 124^{\circ} 53^{\prime} 48^{\prime \prime}}{\sin 14^{\circ} 10^{\prime} 03^{\prime \prime}}=191.778 \\
& \frac{57.23}{\sin 14^{\circ} 10^{\prime} 03^{\prime \prime}}=\frac{C D^{\prime \prime}}{\sin 26^{\circ} 46^{\prime} 06^{\prime \prime}} \\
& C D^{\prime \prime}=\frac{57.23 \cdot \sin 26^{\circ} 46^{\prime} 06^{\prime \prime}}{\sin 14^{\circ} 10^{\prime} 03^{\prime \prime}}=105.310
\end{aligned}
$$

The azimuth of the $A D$ side (again, two possibilities) can be computed, so:

$$
\begin{aligned}
& A D^{\prime}=90^{\circ} 25^{\prime} 53^{\prime \prime}+124^{\circ} 53^{\prime} 48^{\prime \prime}=215^{\circ} 19^{\prime} 41^{\prime \prime} \text { or } 35^{\circ} 19^{\prime} 41^{\prime \prime} \\
& A D^{\prime \prime}=90^{\circ} 25^{\prime} 53^{\prime \prime}+26^{\circ} 46^{\prime} 06^{\prime \prime}=117^{\circ} 11^{\prime} 59^{\prime \prime} \text { or } 297^{\circ} 11^{\prime} 59^{\prime \prime}
\end{aligned}
$$

Which solution is correct is a matter for decision based on other information about the situation.

## A Slightly Different Situation

In the above case, the length of the AC line (the resultant vector) was greater than the length of the known side (AD). A slightly different situation occurs when the length of the AC line is less than the length of the known side. An example of this is shown in Figure 6.38.

The angle at $\mathrm{C}\left(\mathrm{ACD}^{\prime}\right)$ is $302^{\circ} 07^{\prime} 33^{\prime \prime}-278^{\circ} 19^{\prime} 40^{\prime \prime}=23^{\circ} 47^{\prime} 53^{\prime \prime}$. Use the sine rule to compute the angle at D , thus:

$$
\begin{aligned}
& \frac{\sin 23^{\circ} 47^{\prime} 53^{\prime \prime}}{25.13}=\frac{\sin D^{\prime}}{14.916} \\
& \sin D^{\prime}=\frac{14.916 \cdot \sin 23^{\circ} 47^{\prime} 53^{\prime \prime}}{25.13}=0.239507301 \\
& D^{\prime}=13^{\circ} 51^{\prime} 27^{\prime \prime}
\end{aligned}
$$

In this case, the alternate solution of $D^{\prime}=166^{\circ} 08^{\prime} 33^{\prime \prime}$ is not acceptable, because the sum of this angle and the angle at C, $23^{\circ} 47^{\prime} 53^{\prime \prime}$, is greater than $180^{\circ}$.


The azimuth of the line $A D^{\prime}$ is therefore $98^{\circ} 19^{\prime} 40^{\prime \prime}-13^{\circ} 51^{\prime} 27^{\prime \prime}=84^{\circ} 28^{\prime} 13^{\prime \prime}$ and using the sine rule gives 38.047 for the length of $C D^{\prime}$.

Solving the other possible solution, the angle at C (ACD") is $180^{\circ}-23^{\circ} 47^{\prime} 53^{\prime \prime}=156^{\circ} 12^{\prime} 07^{\prime \prime}$. Using the sine rule to compute the angle at $\mathrm{D}^{\prime \prime}$, thus:

$$
\begin{aligned}
& \frac{\sin 156^{\circ} 12^{\prime} 07^{\prime \prime}}{25.13}=\frac{\sin D^{\prime \prime}}{14.916} \\
& \sin D^{\prime \prime}=\frac{14.916 \cdot \sin 156^{\circ} 12^{\prime} 07^{\prime \prime}}{25.13}=0.239507301
\end{aligned}
$$

$$
D^{\prime \prime}=13^{\circ} 51^{\prime} 27^{\prime \prime}
$$

This is the case because the $\mathrm{AD}^{\prime} \mathrm{D}^{\prime \prime}$ triangle is isosceles, as was noted in passing in the previous case. The azimuth of the $A D^{\prime \prime}$ line is therefore $278^{\circ} 19^{\prime} 40^{\prime \prime}+13^{\circ} 51^{\prime} 27^{\prime \prime}=292^{\circ} 11^{\prime} 07 "$. Using the sine rule to compute the length of the $C D^{\prime \prime}$ line, that length is found to be 10.751 .

Note that the calculator program presents the length of the line in the second solution as -10.751 . This is because the sense of the CD line is $278^{\circ} 19^{\prime} 40^{\prime \prime}$, while the actual CD azimuth is the opposite azimuth, $88^{\circ} 19^{\prime} 40^{\prime \prime}$. This warns the user that the triangle forms the other way, i.e., in the opposite direction to the azimuth entered.

Again, it is up to the user to determine which solution is the preferred one for the particular problem under consideration.

Note that all misclosure (errors) in the known part of the traverse will be included in the results of the unknown sides. The resulting traverse should close perfectly, but this is meaningless information as far as the traverse is concerned, as there are no redundant data to allow computation of a misclosure. It is a sensible move to check the closure to make sure it is zero. This is a simple check for data entry errors.

## Running the Program

Begin by running all the fully-known sides through the Closure 1 traverse routine, which runs from Label A. The order of entry of the fully-known sides doesn't matter.

Once all the fully-known sides have been entered, key in the azimuth of the line whose distance is unknown, using HP notation (DDD.MMSS). Press XEQ N.

The program shows ENTER DIST briefly, then prompts for a value for the distance. D?
Key in the distance of the side whose azimuth is unknown. Press R/S.
The program runs for moment, then displays SLN 1 LENG briefly. The calculator then displays the length of the line whose azimuth was known, using $\mathrm{L}=$ in the display.

When this value has been recorded, press R/S.
The calculator displays SLN 1 AZ briefly, then displays the azimuth of the line whose length was known, using $\mathrm{Q}=$ in the display. The azimuth is in HP notation (DDD.MMSSss).

This is the first solution. The second solution follows immediately. Press R/S.
The calculator displays SLN 2 LENG briefly, then displays the length of the line whose azimuth was known, using $\mathrm{L}=$ in the display.

When this value has been recorded, press R/S.
The calculator displays SLN 2 AZ briefly, then displays the azimuth of the line whose length was known, using $\mathrm{Q}=$ in the display. The azimuth is in HP notation (DDD.MMSSss).

This completes the second solution. Press $\mathrm{R} / \mathrm{S}$ to clear the registers used and end the program.

## Sample Computations

## Case 1, from above

| Azimuth | Distance |
| :---: | :---: |
| $61^{\circ} 13^{\prime} 30^{\prime \prime}$ | 90.73 |
| $121^{\circ} 19^{\prime} 10^{\prime \prime}$ | 86.24 |
| $256^{\circ} 15^{\prime} 50^{\prime \prime}$ | Missing distance |
| Missing azimuth | 57.23 |

## Results

Solution 1 Missing distance $=191.778$
Missing azimuth $=35^{\circ} 19^{\prime} 41^{\prime \prime}$

Solution 2 Missing distance $=105.310$
Missing azimuth $=297^{\circ} 11^{\prime} 59^{\prime \prime}$
The choice of solution depends upon other factors, so that is up to the user.

## Case 2, from above

| Azimuth | Distance |
| :---: | :---: |
| $50^{\circ} 12^{\prime} 30^{\prime \prime}$ | 12.81 |
| $170^{\circ} 11^{\prime} 20^{\prime \prime}$ | 16.37 |

$278^{\circ} 19^{\prime} 40^{\prime \prime} \quad$ Missing distance

Missing azimuth
25.13

## Results

Solution 1 Missing distance $=38.047$
Missing azimuth $=84^{\circ} 28^{\prime} 12^{\prime \prime}$

Solution 2 Missing distance $\quad=-10.751$ (Note negative distance, as line is $98^{\circ} 19^{\prime} 40^{\prime \prime}$ )
Missing azimuth $=292^{\circ} 11^{\prime} 08^{\prime \prime}$

## Storage Registers Used

A Azimuth of the partly-known line. Cleared at end.
B Intermediate angle result. Cleared at end.
C 180. Cleared at end.
D Distance input. Cleared at end.
L Solution lengths for display. Cleared at end.
Q Solution azimuths for display. Cleared at end.
Plus those used by the Traverse Closure and Area program (A), including the:
Statistical Registers: $\quad \Sigma \mathrm{x}=$ Current $\Delta \mathrm{Y}$ or $\Delta \mathrm{N}$ from starting point
$\Sigma \mathrm{y}=$ Current $\Delta \mathrm{X}$ or $\Delta \mathrm{E}$ from starting point
$\mathrm{n}=$ Number of sides entered from start

## Labels Used

Label N

$$
\text { Length }=277 \quad \text { Checksum }=77 \mathrm{EA}
$$

Use the length ( $\mathrm{LN}=$ ) and Checksum $(\mathrm{CK}=$ ) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

